#24: $2\pi\int_0^{\pi} \underbrace{\sin x \sqrt{1+\cos^2 x}}_{g(x)}\, dx$. let's call this integral I for later reference. The Simpson rule with the same nodes suggests:

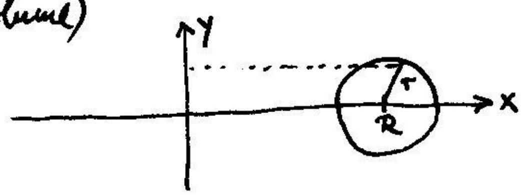
$$I \approx 2\pi \cdot \frac{1}{3} \cdot \frac{\pi}{6} \left(1\cdot g(0) + 4g\left(\tfrac{\pi}{6}\right) + 2g\left(\tfrac{\pi}{3}\right) + 4g\left(\tfrac{\pi}{2}\right) + 2g\left(\tfrac{2\pi}{3}\right) + 4g\left(\tfrac{5\pi}{6}\right) + g(\pi) \right)$$

$$= \frac{2\pi^2}{18}\left(0 + 4\cdot\frac{1}{2}\sqrt{\frac{7}{4}} + 2\,\frac{\sqrt{3}}{2}\sqrt{\frac{5}{4}} + 4\cdot 1\sqrt{1} + \dots \right)$$

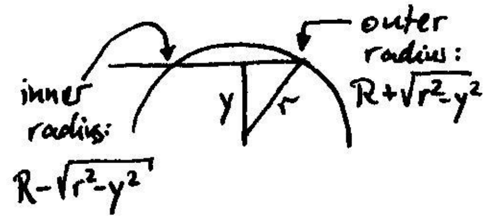
$$= \frac{\pi^2}{9}\left(2\sqrt{7} + \sqrt{15} + 4\right) \approx 14.4365$$

(Numerics was not required)

#25: (Volume)



(zoom:



$$\text{Volume} = \pi\int_{-r}^{r}\left(\left(R+\sqrt{r^2-y^2}\right)^2 - \left(R-\sqrt{r^2-y^2}\right)^2\right)dy \;=\; \text{symmetry upper–lower}$$

$$= 2\pi\int_0^r 4R\sqrt{r^2-y^2}\,dy = 8\pi R \underbrace{\int_0^r \sqrt{r^2-y^2}\,dy}_{\frac{\pi}{4}r^2 \text{ : area of } \frac{1}{4} \text{ disc}}$$

(or: see below)

$$= \underline{\underline{2\pi^2 R r^2}}$$

If we want the "outer half" and "inner half" separately, we must work a bit harder:

$$V_{outer} = 2\pi\int_0^r\left(\left(R+\sqrt{r^2-y^2}\right)^2 - R^2\right)dy = 2\pi\int_0^r 2R\sqrt{r^2-y^2}\,dy + 2\pi\int_0^r (r^2-y^2)\,dy$$