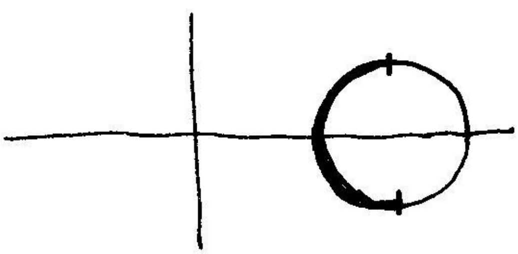
**Note:** formal integration of $\int_{-r}^{r} \sqrt{r^2 - x^2}\, dx$ via substitution

$$x = r \sin\varphi \qquad dx = r \cos\varphi \, d\varphi \qquad :$$

$$\int_{-r}^{r} \sqrt{r^2 - x^2}\, dx = \int_{-\pi/2}^{\pi/2} r^2 \cos^2\varphi \, d\varphi = r^2 \int_{-\pi/2}^{\pi/2} \frac{1 + \cos 2\varphi}{2}\, d\varphi$$

$$= r^2 \left[ \frac{1}{2}\varphi + \frac{1}{4} \sin 2\varphi \right]_{-\pi/2}^{\pi/2} = \underline{\underline{\frac{\pi}{2} r^2}}$$

**#25:** (Surface area)



Note that we are rotating about the y axis, not about the x-axis.
The curves are therefore to be written as $x = f(y)$

namely: $\underbrace{x = R + \sqrt{r^2 - y^2}}_{\text{outer curve}}$ and $x = \underbrace{R - \sqrt{r^2 - y^2}}_{\text{inner curve}}$

We automatically get the "outer" and "inner" half of the surface separately

outer half of surface: $$2\pi \int_{-r}^{r} \left(R + \sqrt{r^2 - y^2}\right) \sqrt{1 + \underbrace{\left(\frac{dx}{dy}\right)^2}_{\left(\frac{-y}{\sqrt{r^2 - y^2}}\right)^2}}\; dy =$$

$$= 2\pi \int_{-r}^{r} \left(R + \sqrt{r^2 - y^2}\right) \frac{r}{\sqrt{r^2 - y^2}}\, dy$$

$$= 2\pi R \int_{-r}^{r} \frac{r\, dy}{\sqrt{r^2 - y^2}} + \underbrace{2\pi \cancel{R}\, r \int_{-r}^{r} dy}_{\to\ 4\pi r^2}$$

~~Subst. letter~~ $y = R\sin$