#31 cont'd: (a) in analogy for the double-angle trick with trig-fcts;

(b) by writing $\cosh t$ in terms of exponentials;

We pursue both:

(a) $\int \cosh^2 t \, dt = \int \frac{\cosh 2t + 1}{2} \, dt = \frac{1}{2} t + \frac{1}{4} \sinh 2t + C.$

(b) $\int \cosh^2 t \, dt = \int \left(\frac{e^t + e^{-t}}{2}\right) dt = \int \frac{1}{4}\left(e^{2t} + 2 + e^{-2t}\right) =$

$= \frac{1}{2} t + \frac{1}{8} e^{2t} - \frac{1}{8} e^{-2t} + C$

$= \frac{1}{2} t + \frac{1}{4} \sinh 2t + C$

We'll later discover a remarkable relation also between trigs & exponentials, which will give (kind of) an analog to (b) also for $\int \cos^2 t \, dt$.

Anyways, $\text{area} = \ldots = \frac{2\pi}{a^2} \int_{-ab}^{ab} \cosh^2 t \, dt = \underline{\underline{\frac{2\pi}{a^2}\left(ab + \frac{1}{2} \sinh 2ab\right)}}$

#32: We are dealing with $I = 2\pi I_0$

$$\underline{\underline{I_0 = \int_0^{\pi} \sin x \sqrt{1 + \cos^2 x} \, dx}}$$

(a) With $s = \cos x$, $ds = -\sin x \, dx$, we get

$$I_0 = -\int_1^{-1} \sqrt{1 + s^2} \, ds = \underline{\underline{\int_{-1}^{1} \sqrt{1 + s^2} \, ds}}$$

(b1) $s = \tan u$, $ds = \frac{du}{\cos^2 u}$, $\sqrt{1 + s^2} = \sqrt{1 + \tan^2 u} = \frac{1}{\cos u}$

$$\underline{\underline{I_0 = \int_{-\pi/4}^{\pi/4} \frac{du}{\cos^3 u}}}$$