#32: (b2) $\quad s = \sinh v \qquad \sqrt{1+s^2} = \cosh v \qquad ds = \cosh v\, dv$

$$I_0 = \int_{-\operatorname{arsinh} 1}^{\operatorname{arsinh} 1} \cosh^2 v\, dv$$

Here, arsinh ~~1~~ is the inverse fct of sinh (which we haven't really discussed yet). But we can avoid it: we want to find v such that ~~e^v~~ $\frac{e^v - e^{-v}}{2} = 1$, or $e^{2v} - 2e^v - 1 = 0$.

This is a quadratic eqn in e^v; it sol'n is

$$e^v = 1 \pm \sqrt{1+1} = 1 + \sqrt{2} \qquad (\text{discard minus b/c } e^v > 0)$$

$$\underline{\underline{I_0 = \int_{-\ln(1+\sqrt{2})}^{\ln(1+\sqrt{2})} \cosh^2 v\, dv}} = 2\int_0^{\ln(1+\sqrt{2})} \cosh^2 v\, dv$$

#33: (b2) we know this trick already (see #31)

$$I_0 = \left[v + \tfrac{1}{2}\sinh 2v\right]_0^{\ln(1+\sqrt{2})} = \underline{\underline{\ln(1+\sqrt{2}) + \tfrac{1}{2}\sinh(2\ln(1+\sqrt{2}))}}$$

Writing sinh in terms of exp, we can simplify this a bit:

$$\tfrac{1}{2}\sinh 2\ln(1+\sqrt{2}) = \frac{\exp(2\ln(1+\sqrt{2})) - \exp(-2\ln(1+\sqrt{2}))}{4} =$$

$$= \frac{(1+\sqrt{2})^2 - (1+\sqrt{2})^{-2}}{4} = \frac{(\sqrt{2}+1)^2 - (\sqrt{2}-1)^2}{4} = \underline{\underline{\sqrt{2}}}$$

$$\underline{\underline{I_0 = \ln(1+\sqrt{2}) + \sqrt{2}}}$$

(numerically: $I = 2\pi I_0 = 14.4236$)