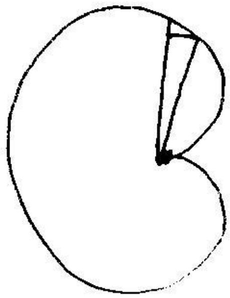
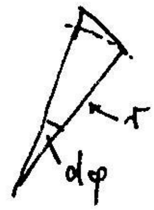
#36: (As sketched in class,) we first need to find ~~area~~ the formula for the area



The area is a limit of a Riemann sum whose terms representing small sectors :



The area of a sector with angle α is $\frac{1}{2}\alpha r^2$ (pro-rated from πr^2 for $\alpha = 2\pi$).

Here α is $d\varphi$

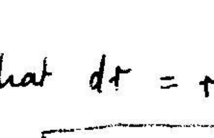
$$\boxed{\text{area} = \frac{1}{2}\int_0^{2\pi} r(\varphi)^2\, d\varphi} = \frac{1}{2}\int_0^{2\pi} 4\,(1-\cos\varphi)^2 d\varphi =$$

$$= 2\int_0^{2\pi} (1 - 2\cos\varphi + \cos^2\varphi)\, d\varphi = 2\left[\varphi - 2\sin\varphi + \frac{1}{2}\varphi + \frac{1}{2}\sin\varphi\cos\varphi\right]_0^{2\pi}$$

Noting $\cos^2\varphi = \frac{1}{2}(1+\cos 2\varphi)$

$$= \underline{6\pi}$$

To find the length, we need to find an appropriate formula again:



(piece of a spherical arc; as good as a short straight piece for the purpose of Riemann sums for the length)

Note that $dr = r'(\varphi)\, d\varphi$

$$ds^2 = \sqrt{(dr)^2 + (r\,d\varphi)^2} = \sqrt{r'^2 + r^2}\, d\varphi$$