Therefore, in polar coordinates,

$$L = \int_0^{2\pi} \sqrt{r^2 + r'^2}\, d\varphi = \int_0^{2\pi} 2\sqrt{(1-\cos\varphi)^2 + \sin^2\varphi}\; d\varphi =$$

using the cardioid formula
$r(\varphi) = 2(1-\cos\varphi)$
$r'(\varphi) = 2\sin\varphi$

$$= 2\int_0^{2\pi} \sqrt{1 - 2\cos\varphi + \cos^2\varphi + \sin^2\varphi}\; d\varphi = 2\int_0^{2\pi} \sqrt{2 - 2\cos\varphi}\; d\varphi$$

$$= 2\int_0^{2\pi} \sqrt{4\sin^2\frac{\varphi}{2}}\; d\varphi = 4\int_0^{2\pi} \sin\frac{\varphi}{2}\, d\varphi \underset{\psi = \frac{\varphi}{2}}{=} 8\int_0^{\pi} \sin\psi\, d\psi$$

$$= \underline{\underline{16}}$$

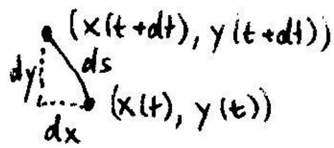
Test with isoperimetric inequality:

A disk with the same area (6π) as the cardioid has radius $\sqrt{6}$ and therefore perimeter $2\sqrt{6}\,\pi$. This should be < 16.

Indeed, $2\sqrt{6}\,\pi \approx 15.39 < 16$

So our results are plausible.

Next, let's recalculate the length from the parametric form (trusting that we are still talking about the same curve, as the professor claims we are):



$$\left.\begin{aligned} dy &= y'(t)\,dt \\ dx &= x'(t)\,dt \end{aligned}\right\} \Rightarrow ds = \sqrt{dx^2 + dy^2} = \sqrt{x'^2 + y'^2}\; dt$$