So we obtain $L = \int_{t_0}^{t_1} \sqrt{(-2\sin t + 2\sin 2t)^2 + (2\cos t - 2\cos 2t)^2}\, dt$

$$= \int_{t_0}^{t_1} \sqrt{4\sin^2 t + 4\sin^2 2t - 8\sin t \sin 2t + 4\cos^2 t + 4\cos^2 2t - 8\cos t\cos 2t}\, dt$$

$$= \int_{t_0}^{t_1} \sqrt{4+4-8\underbrace{(\cos t\cos 2t + \sin t\sin 2t)}_{=\cos(2t-t) \text{ by addition theorem}}}\, dt$$

$$= \int_{t_0}^{t_1} \sqrt{8-8\cos t}\, dt = \int_{t_0}^{t_1} \sqrt{4\cdot 4\sin^2\tfrac{t}{2}} = 4\int_{t_0}^{t_1} \left|\sin\tfrac{t}{2}\right| dt$$

Hey, we weren't told what t_0, t_1 are; but we can find out. We want to go once around. After t increases by 2π, both $x(t)$ and $y(t)$ are the same again, and no earlier. So any interval of length 2π should be an appropriate range for t. For instance $t_0 = 0$, $t_1 = 2\pi$. And we're back at the old integral again:

$$4\int_0^{2\pi} \left|\sin\tfrac{t}{2}\right| dt = 4\int_0^{2\pi} \sin\frac{t}{2}\, dt = 8\int_0^{\pi} \sin\tau\, d\tau = \underline{16}$$

<u>#37</u>: Let's first get a sketch of the curve $x^{2/3} + y^{2/3} = 1$

Clearly (set $y=0$) the x-intercepts are ± 1, and similarly (set $x=0$) the y-intercepts are ± 1. $\quad y = \pm\sqrt{(1-x^{2/3})^3}$

The figure is symm. wrt the y axis (since ~~nothin~~ the change $x \to -x$ gives the same formula again), symm. wrt the x axis (changing $y \to -y$ gives same formula); and also symmetric wrt the diagonal $x = y$, because