

$$\left. \begin{aligned} \text{perimeter} &= 4 \int_0^1 \sqrt{1+x^{-2/3}-1} \, dx = 4 \int_0^1 x^{-1/3} \, dx \\ &= 4 \left[\frac{3}{2} x^{2/3} \right]_0^1 = 4 \cdot \frac{3}{2} = \underline{\underline{6}} \end{aligned} \right\} (*)$$

Note: The Riemann integral $\int_0^1 x^{-1/3} \, dx$ does not exist since

$x^{-1/3}$ is unbounded as $x \rightarrow 0^+$. However, if we take only the length between $x = \varepsilon$ and $x = 1$ and then take $\lim_{\varepsilon \rightarrow 0}$, we get basically the same result in a cleaner way:

$$\left. \begin{aligned} \text{perimeter} &= 4 \lim_{\varepsilon \rightarrow 0} \int_{\varepsilon}^1 \sqrt{1+x^{-2/3}-1} \, dx = 4 \lim_{\varepsilon \rightarrow 0} \int_{\varepsilon}^1 x^{-1/3} \, dx = \\ &= 4 \lim_{\varepsilon \rightarrow 0} \left[\frac{3}{2} x^{2/3} \right]_{\varepsilon}^1 = 6 - \lim_{\varepsilon \rightarrow 0} 6 \varepsilon^{2/3} = \underline{\underline{6}}. \end{aligned} \right\} (**)$$

The above calculation (*) should be read as a shorthand for the precise calculation (**).