#41:

$$I_n = \int_0^{\pi/2} \sin^n x \, dx = \int_0^{\pi/2} \sin^{n-1} x \cdot \sin x \, dx =$$

$$(n-1)\sin^{n-2}x \cdot \cos x \qquad -\cos x$$

$$= \underbrace{\Big[-\cos x \sin^{n-1} x\Big]_0^{\pi/2}}_{=0} + \int_0^{\pi/2} (n-1) \sin^{n-2} x \cos^2 x \, dx$$

$$= (n-1) \int_0^{\pi/2} \sin^{n-2} x \, (1-\sin^2 x) \, dx =$$

$$= (n-1)\left(I_{n-2} - I_n\right)$$

$$\underline{\underline{I_n = \frac{n-1}{n} I_{n-2}}}$$

$$I_1 = \int_0^{\pi/2} \sin x \, dx = 1$$

therefore $I_3 = \frac{2}{3} \cdot I_1 = \frac{2}{3}$

$$I_5 = \frac{4}{5} \cdot \frac{2}{3}$$

$$I_7 = \frac{6}{7} \cdot \frac{4}{5} \cdot \frac{2}{3}$$

$$I_n = \frac{(n-1)(n-3)\cdot \ldots \cdot 2}{n \quad (n-2) \cdot \ldots \cdot 3} \quad \text{for } n \ \underline{\text{odd}}.$$

$$I_2 = \int_0^{\pi/2} \sin^2 x \, dx = \ldots = \frac{\pi}{4}$$

$$I_4 = \frac{3}{4} I_2 = \frac{3}{4} \cdot \frac{\pi}{4}$$

$$I_6 = \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{\pi}{4}$$

$$I_n = \frac{(n-1)(n-3)\cdot \ldots \cdot 3}{n \quad (n-2) \cdot \ldots \cdot 4} \cdot \frac{\pi}{4} \quad \text{for } n \ \underline{\text{even}}$$