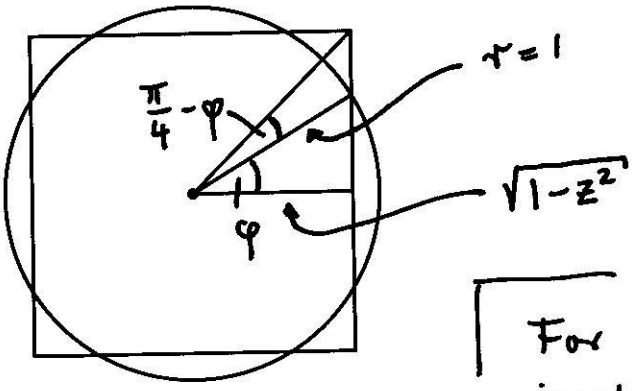
#38: At level z (with $-1 \le z \le 1$) we again depict the cross section of the body under investigation.

The cylinders from #34 intersect this plane in two stripes, of which the intersection is a square of sidelength $2 \cdot \sqrt{1-z^2}$

From the third cylinder, we get a circle of radius 1.

The cross section at level z is the intersection of circle and square: The figure is for $|z| < \frac{\sqrt{2}}{2}$.



$\cos\varphi = \sqrt{1-z^2}$

$\sin\varphi = z$

$\varphi = \arcsin z$

For $|z| > \frac{\sqrt{2}}{2}$, the square is all inside the circle

For $|z| < \frac{\sqrt{2}}{2}$, the cross section consists of

- four sectors, each with opening angle $2 \cdot (\frac{\pi}{4} - \varphi) = (\frac{\pi}{2} - 2\varphi)$
  (their total area is $\frac{1}{2} \cdot 1^2 \cdot 4 \cdot (\frac{\pi}{2} - 2\varphi)$)
- four triangles, each with area $2 \cdot \frac{1}{2} z \sqrt{1-z^2}$
  (their total area is $4z\sqrt{1-z^2}$)

For $|z| \ge \frac{\sqrt{2}}{2}$, the cross section is a square of area $4(1-z^2)$

$$\text{Total Volume} = 2\int_0^{\sqrt{2}/2} \left(\pi - 4\underbrace{\varphi}_{=\arcsin z} + 4z\sqrt{1-z^2}\right) dz + 2\int_{\sqrt{2}/2}^{1} 4(1-z^2)\, dz$$