To prepare for this substitution, we have to split the integral

$$M(a,b) = 2\int_0^{\infty} \frac{dx}{\sqrt{(x^2+a^2)(x^2+b^2)}} = 2\int_{-\infty}^{\infty} \frac{1+\frac{u}{\sqrt{u^2+ab}}}{\sqrt{\left[\left(u+\sqrt{u^2+ab}\right)^2+a^2\right]\left[\dots +b^2\right]}}\,du$$

$$x = u+\sqrt{u^2+ab}$$

$$dx = \left(1+\frac{u}{\sqrt{u^2+ab}}\right)du$$

If $x \to 0$, then $u \to -\infty$

If $x \to +\infty$, then $u \to +\infty$

Let's clean up the mess under the integral. The "prophecy" that $M(a,b)$ will be equal to $M\left(\sqrt{ab}, \frac{a+b}{2}\right)$ will guide us. And we already see the $\frac{1}{\sqrt{u^2+ab}}$ as our friend, because this shows up already.

Under the big $\sqrt{}$ in the denominator we have

$$\left[\left(u+\sqrt{u^2+ab}\right)^2+a^2\right]\left[\left(u+\sqrt{u^2+ab}\right)^2+b^2\right] =$$

$$= \left[u^2+u^2+ab+2u\sqrt{u^2+ab}+a^2\right]\left[u^2+u^2+ab+2u\sqrt{u^2+ab}+b^2\right] =$$

$$= \left(2u^2+2u\sqrt{u^2+ab}+a(a+b)\right)\left(2u^2+2u\sqrt{u^2+ab}+b(a+b)\right) =$$

$$= 4u^4+8u^3\sqrt{u^2+ab}+4u^2(u^2+ab)+2u^2\left(b(a+b)+a(a+b)\right)$$
$$+\,2u\sqrt{u^2+ab}\,\left(b(a+b)+a(a+b)\right)+ab(a+b)^2$$

$$= 4u^2\left(u+\sqrt{u^2+ab}\right)^2+2u\left(u+\sqrt{u^2+ab}\right)(a+b)^2+ab(a+b)^2$$