#44:

$$\sin x < x \qquad (\text{for } x>0)$$

$$\int_0^x \sin t\,dt < \int_0^x t\,dt$$

$$1-\cos x < \frac{x^2}{2}$$

hence

$$\cos x > 1-\frac{x^2}{2}$$

$$\int_0^x \cos t\,dt > \int_0^x \left(1-\frac{t^2}{2}\right)dt$$

$$\sin x > x - \frac{x^3}{3\cdot 2}$$

$$\int_0^x \sin t > \int_0^x \left(t - \frac{t^3}{3\cdot 2}\right)dt$$

$$1-\cos x > \frac{x^2}{2} - \frac{x^4}{4\cdot 3\cdot 2}$$

hence

$$\cos x < 1 - \frac{x^2}{2} + \frac{x^4}{4\cdot 3\cdot 2}$$

etc.

$$\cancel{\text{So} > x > \frac{x^3}{3\cdot 2} + > \frac{x^5}{5!} > -\frac{x^7}{7!} + -}$$

So $\sin x < x$

$$\text{-"-} \quad > x - \frac{x^3}{3!}$$

$$\text{-"-} \quad < x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

$$\text{-"-} \quad > x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$$

etc

For $x<0$, these inequalities are reversed.

$$\cos x > 1 - \frac{x^2}{2!}$$

$$\cos x < 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

$$\cos x > 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$$

$$\vdots$$

These are also true for $x<0$, since $\cos(-x) = \cos x$