Note that the denom., as given is not factored yet completely:
The (x^2-1) term can be factored as $(x-1)(x+1)$

We thus get the PFD

$$x+4+\frac{3x^6+4x^5+20x^4+4x^3+22x^2+x-18}{(x^2+2)^2(x-1)(x+1)(x+3)}=$$

$$=x+4+\frac{ax+b}{(x^2+2)^2}+\frac{cx+d}{x^2+2}+\frac{e}{x-1}+\frac{f}{x+1}+\frac{g}{x+3}$$

Here is a little trick how to evaluate a polynomial efficiently for particular values of x. Write (eg.)

$$3x^6+4x^5+20x^4+4x^3+22x^2+x-18=$$

$$\left\{\left[\left(\left(\left(3x+4\right)x+20\right)x+4\right)x+22\right]x+1\right\}x-18$$

For $x=1$: $\quad$ ~~66~~ 36

For $x=-1$: $\quad$ ~~54~~ 18

For $x=-3$: $\quad -5 \rightarrow$ $\quad -105 \rightarrow$ $\quad 325 \rightarrow$ $\quad -974 \rightarrow$

$3\cdot 974-18=$ ~~2940~~ 2904

Cover-up method for e, f, g:

$$e=\frac{36}{3^2\cdot 2\cdot 4}=\frac{1}{2}$$

$$f=\frac{18}{3^2\cdot(-2)(2)}=-\frac{1}{2}$$

$$g=\frac{2904}{11^2\cdot(-4)(-2)}=3$$

Cover-up method for a, b involves $x\rightarrow\sqrt{2}\,i$