$$a\sqrt{2}\,i + b = \frac{\text{Numerator at } \sqrt{2}\,i}{(-3)(3+\sqrt{2}\,i)}$$

For the evaluation of the numerator, I do even and odd powers separately b/c that way I get the real and imaginary parts sorted right away.

$$3x^6 + 20x^4 + 22x^2 - 18 \,\Big|_{x=\sqrt{2}\,i} = \Big[\big(3(-2)+20\big)(-2)+22\Big](-2)-18$$

$$= \underline{\underline{-6}}$$

$$x\,(4x^4+4x^2+1)\,\Big|_{x=\sqrt{2}\,i} = \sqrt{2}\,i\Big(\big(4(-2)+4\big)(-2)+1\Big) = \underline{9\sqrt{2}\,i}$$

$$a\sqrt{2}\,i + b = \frac{-6+9\sqrt{2}\,i}{(-3)(3+\sqrt{2}\,i)} = \frac{(2-3\sqrt{2}\,i)(3-\sqrt{2}\,i)}{(3+\sqrt{2}\,i)(3-\sqrt{2}\,i)} = \frac{0-11\sqrt{2}\,i}{9+2} = \underline{\underline{-\sqrt{2}\,i}}$$

$$\underline{\underline{a=-1\,,\ b=0}}$$

$$\frac{3x^6+4x^5+20x^4+4x^3+22x^2+x-18}{(x^2+2)^2\,(x^2-1)(x+3)} + \frac{x\,(x^2-1)(x+3)}{(x^2+2)^2\,(x^2-1)(x+3)} =$$

$$= \frac{cx+d}{x^2+2} + \frac{e}{x-1} + \frac{f}{x+1} + \frac{g}{x+3}$$

Common numerator on left side:

$$3x^6+4x^5+21x^4+7x^3+21x^2-2x-18 = (x^2+2)(3x^4+4x^3+15x^2-x-9)$$

So we need c, d to satisfy

$$\frac{3x^4+4x^3+15x^2-x-9}{(x^2+2)(x^2-1)(x+3)} = \frac{cx+d}{x^2+2} + \frac{e}{x-1} + \frac{f}{x+1} + \frac{g}{x+3}$$

$$\underbrace{\frac{12-30-9+\sqrt{2}\,i(-8-1)}{(-3)(3+\sqrt{2}\,i)}}_{=3} = c\sqrt{2}\,i + d \qquad \underline{\underline{c=0\,,\ d=3}}$$