

We have thus found the pfd.

$$x+4 + \frac{-x}{(x^2+2)^2} + \frac{3}{x^2+2} + \frac{1/2}{x-1} - \frac{1/2}{x+1} + \frac{3}{x+3}$$

I'll check the calculation at $x=0$ and $x=2$:

$$\underline{x=0:} \quad \frac{-66}{-12} \stackrel{?}{=} 4 + \frac{3}{2} - \frac{1}{2} - \frac{1}{2} + 1 \quad \frac{11}{2} = \frac{11}{2} \quad \checkmark$$

$$\underline{x=2:} \quad \frac{3984}{6^2 \cdot 3 \cdot 5} \stackrel{?}{=} 6 - \frac{2}{36} + \frac{3}{6} + \frac{1}{2} - \frac{1}{6} + \frac{3}{5}$$

$$7.3777... \stackrel{?}{=} 7.3777... \quad \checkmark$$

We can now immediately find an antiderivative

$$\frac{x^2}{2} + 4x + \frac{1/2}{x^2+2} + \frac{3}{\sqrt{2}} \arctan \frac{x}{\sqrt{2}} + \frac{1}{2} \ln|x-1| - \frac{1}{2} \ln|x+1| + 3 \ln|x+3|$$