UTK – M231 – Differential Equations
Notes on Chapter 3 – Jochen Denzler, Oct 2002

Ok, "Notes" is a bit of an overstatement. Some of you feel still pretty lost with the modelling stuff. I want you to do *one* problem, just from scratch. Not too much of an analog in the book, so you can think about it without bias. Only setting up the equation, you don't need to solve it (you couldn't solve it anyway). You have 3 weeks for it, not because it's so much work, but simply because I want you to think about it without hurry. Come in for discussion, when needed, discuss with your peers, get any help you wish. But in the end, lay down a clearly explained exposition of what you are doing. I'll grade it myself. If I think the exposition is not sufficiently clear, I'll come back and ask questions to get it clarified. This hwk will be worth a maximum of 40 points towards your homework score. So you see, I really mean it. Everybody has a fair chance to make these 40 points, also those of you who believe they can't.

Here is the problem:

A central hot water system in a house operates with a hot water reservoir of volume $V = 1000L$. If tenants take hot water from the faucet, an equal amount of cold water (temperature $20°C$) from the pipes will replace it. We assume that the water in the container gets mixed fast enough so that the temperature of the water in the container is the same everywhere in the container. The hot water usage U (in L/min) is given as a function of time. We won't specify a formula; a plausible graph may look like the one given below.

The heating system has a temperature sensor that sets the power P (in J/min) automatically in dependence of the temperature T. Again, we don't specify a formula; a plausible P is graphed below.

It takes 4.2×10^3 J to heat one liter of water by $1\ °C$. The physics of heating tells us that the amount of heat needed for heating is proportional to the quantity of water, and also proportional to the temperature difference the heating is to achieve.

Set up the differential equation for the temperature as a function of time.