

The crescent shaped domain D below is bounded by portions of circles of radius 6 and 4 respectively. Their centers are distance 3 apart. Calculate $\int_D xy^2 dA$. Hints:

(a) While D is neither horizontally nor vertically simple, you can write $D = D_+ \cup D_-$, where D_+ and D_- are the parts of D above and below the x -axis, and they are vertically simple.

Determine functions $g_1(x), g_2(x)$ st. $D_+ = \{(x, y) : g_1(x) \leq y \leq g_2(x)\}$

Note that g_i will be a piecewise function $g_i(x) = \begin{cases} \dots & \text{if } x \leq \dots \\ \dots & \text{if } x \geq \dots \end{cases}$

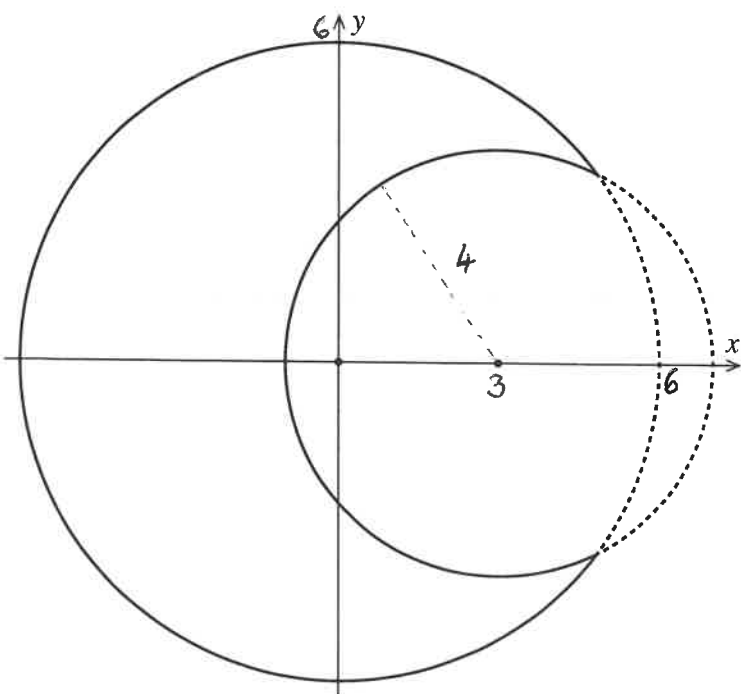
(b) Find out the smallest x (easy)

and the largest x (from intersection of two circles)

to determine the limits of integration in $\int_{?}^{?} \dots dx$

(c) You will split $\int_{?}^{?} \dots dx$ into two pieces

$\int_{?}^{?} \dots dx + \int_{?}^{?} \dots dx$ according to where the formula for g_i changes.



(d) Once you have reduced

$\int_D xy^2 dA$ to single variable integrals $\int_a^b \dots dx + \int_b^c \dots dx$

you are welcome to use symbolic algebra software or numerical methods (Wolfram-Alpha, Mathematica, Maple, Matlab, ...)

to finish it up (even though it can be done by Calc 2 methods if need be)