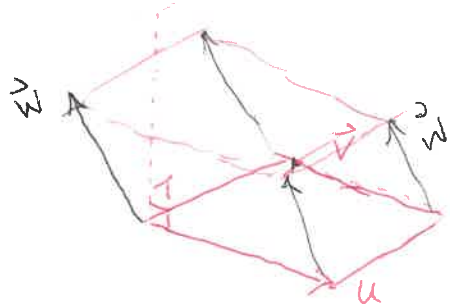
Scalar Triple Product



parallelepiped

area of the base parallelogram:

$\|\vec{u} \times \vec{v}\|$

angle btw base parallelogram and vector $\vec{w}$ is either $90° + \alpha$ or $90° - \alpha$ where α is the angle btw base plane and the normal vector $\vec{u} \times \vec{v}$

$$\text{Height of box} = \|\vec{w}\| \cdot \underbrace{\sin(90° \pm \alpha)}_{\cos\alpha}$$

where $\alpha = \measuredangle(\text{normal } \vec{u} \times \vec{v}, \ 3^{\text{rd}} \text{ vector } \vec{w})$

$$\text{"oriented" Volume} = \overbrace{\|\vec{u} \times \vec{v}\|}^{\text{base area}} \cdot \overbrace{\|\vec{w}\| \cdot \cos\alpha}^{\text{height}}$$

$$= (\vec{u} \times \vec{v}) \cdot \vec{w}$$

this combination is called scalar triple product

$$\text{geometric volume} = |(\vec{u} \times \vec{v}) \cdot \vec{w}|$$

BTW: $(\vec{u} \times \vec{v}) \cdot \vec{w} = (\vec{v} \times \vec{w}) \cdot \vec{u} =$

If $\vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$ etc then

$$= u_1 v_2 w_3 + u_2 v_3 w_1 + u_3 v_1 w_2 - u_2 v_1 w_3 - u_3 v_2 w_1 - u_1 v_3 w_2$$

$$=: \begin{vmatrix} u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_3 & v_3 & w_3 \end{vmatrix}$$ "3x3 determinant"

Sneak preview: ~~1x1 determinants~~

1x1 determinants are nothing but the number itself.

det will show up when you do the analog of u substitution in MV integrals

Instead of $u = f(x) \Rightarrow du = f'(x)\, dx$

we will have $(u_1, u_2, u_3) = f_{1,2,3}(x_1, x_2, x_3)$

$$\text{"}du\text{"} = \det(\text{deriv. of } f)\ \text{"}dx\text{"}$$

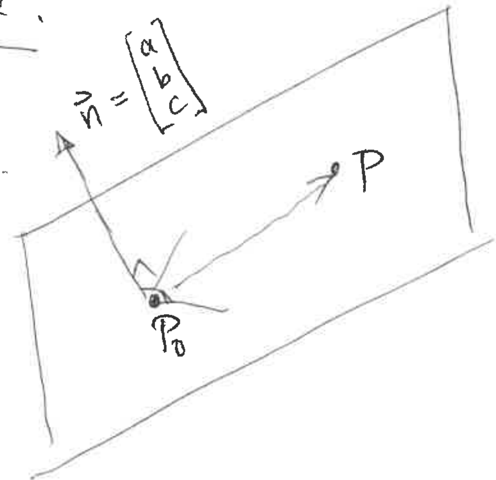
Details later

Planes in $\mathbb{R}^3$

For a pt (x,y,z) to lie on a certain plane, there will be numbers a, b, c, d (not unique) $\begin{bmatrix} a \\ b \\ c \end{bmatrix} \neq \vec{0}$ such that (x,y,z) is on the plane if and only if

$$\boxed{ax+by+cz=d}$$

How come?



P_0 in plane

$P_0(x_0, y_0, z_0)$

$P(x,y,z)$

$$\overrightarrow{P_0P} = \begin{bmatrix} x-x_0 \\ y-y_0 \\ z-z_0 \end{bmatrix}$$

$$\overrightarrow{P_0P} \cdot \vec{n} = 0$$

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

$$ax+by+cz = \underbrace{ax_0+by_0+cz_0}_{d}$$