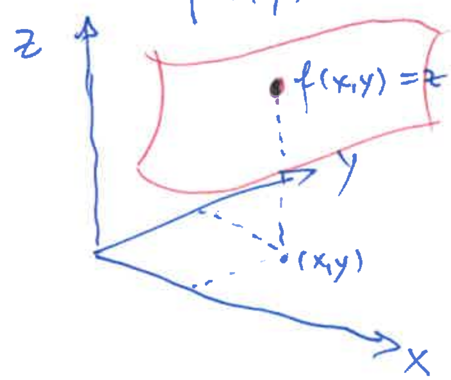
Functions of several variables

(let's stick with 2 variables for the next few pages)

Ex: $f(x,y) = x+y$ "plus" fct

$f(x,y) = x \cdot y$ "times" fct

$f(x,y) = x^2+2y^2$



some f, not among those

many examples in Sec 14.1

The following cuts of such a surface with planes are of significance:

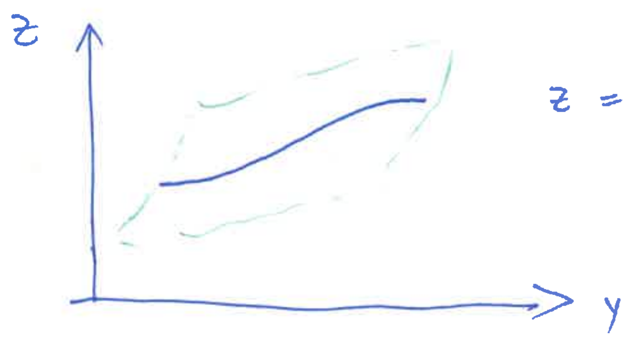
(1) freeze $x=a$, consider the SV fct $y \mapsto f(a,y)$

a.k.a $f(a,\cdot)$

(2) freeze $y=b$, consider SV fct $f(\cdot,b)$ aka

$x \mapsto f(x,b)$

These cuts / slices are obtained by intersecting graph f with a vertical plane $x=a$ or $y=b$ resp'ly



$z = f(a,y)$

(3) Level curves

Curves in the (x,y) plane that satisfy $f(x,y)=c$ with c fixed

Like in hiking maps...

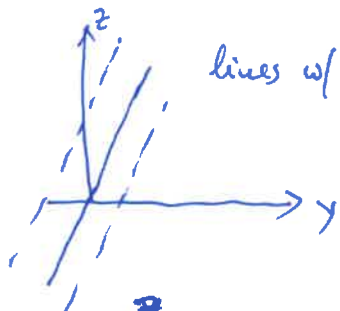
isothermes in a weather map

isobares —"—

Ex: $f(x,y) = x+3y$

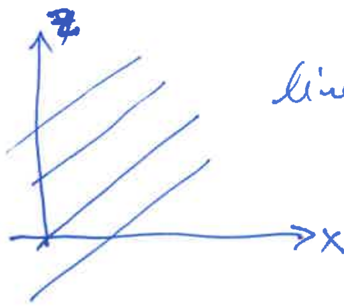
graph f is a plane $z-x-3y=0$

freeze $x=a$



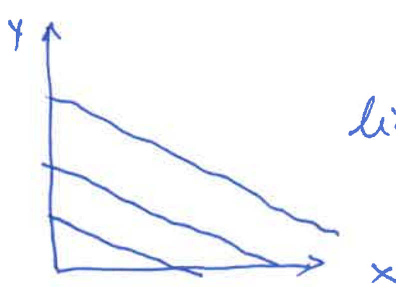
lines w/ slope 3 (which line depends on a)

freeze $y=b$



lines w/ slope 1

level curves : $x+3y=c$



lines w/ slope $-\frac{1}{3}$

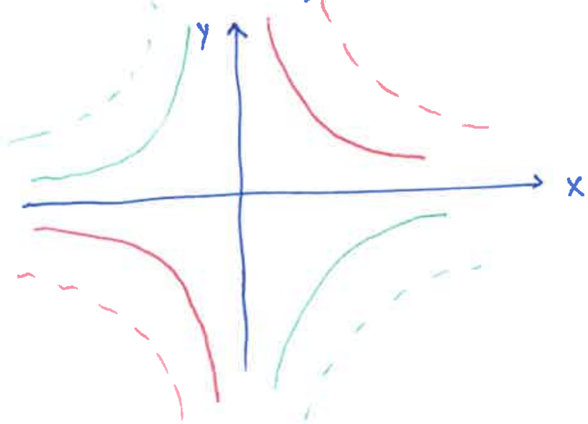
(boring...)

$f(x,y) = x \cdot y$

freeze $x=a$ $\quad z = ay$

freeze $y=b$ $\quad z = bx$

level curves: $xy = c$



level curve @ level $c=0$

level curve @ $c=1$ ———

@ $c=2$ - - - -

level @ $c=-1$ ———

$c=-2$ - - - -

Example:

$$f(x,v) = -\cos x + \frac{1}{2}v^2$$

(Applic: energy map of a "mathematical pendulum"

$-\cos x$ potential energy, $\frac{1}{2}v^2$ kinetic energy)

level curves curves of constant energy

level -1: can only happen if $x \in \{0, \pm 2\pi, \pm 4\pi, \ldots\}$

$v=0$

. . . .

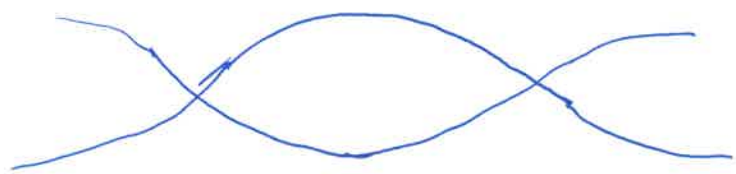
level $-\frac{2}{3}$

$$-\cos x + \frac{1}{2} v^2 = -\frac{2}{3}$$

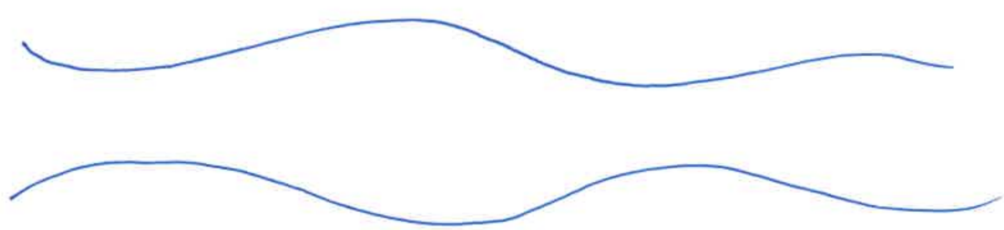
$$v = \pm\sqrt{2\left(\cos x - \frac{2}{3}\right)}$$

similiar all the way up to level 1

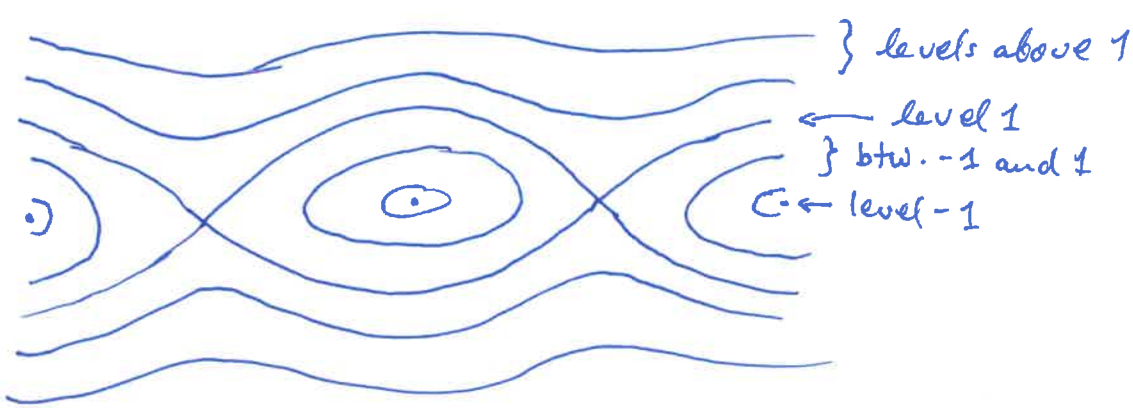
$$v = \pm\sqrt{2(\cos x + 1)} = \pm\sqrt{2 \cdot 2\cos^2 \frac{x}{2}} = \pm 2\cos\frac{x}{2}$$



level above 1



Put them all together:



3 variables:

level sets are typically surfaces

$$f(x,y,z) = x^2+y^2+z^2$$

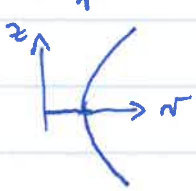
@ pos. levels — spheres
@ level 0 — origin
@ level < 0 — empty set

(Monday already)

Level surfaces for $x^2+y^2-z^2$. (Using polar coord's in (x,y) plane helps.)

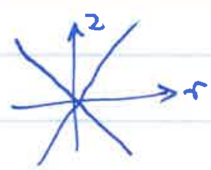
level $c>0$, eg $c=1$:

$$z = \pm\sqrt{\underbrace{x^2+y^2}_{r^2}-1}$$



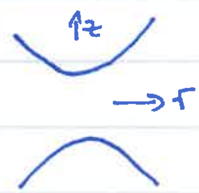
level $c=0$

$$z = \pm\sqrt{x^2+y^2} = \pm r$$

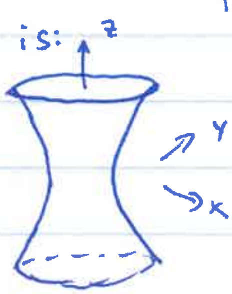


level $c<0$ eg $c=1$

$$z = \pm\sqrt{r^2+1}$$



thus level surface is:



thus level surface

double-cone

thus level surface

these are called hyperboloids of 1 or 2 sheets respectively