Note: Study of this example was simplified by the fact that $f(x,y,z)$ depended on (x,y) only through the combination x^2+y^2, which is r^2 in polar coord's in the (x,y) plane.
The level surfaces of f are therefore symmetric under rotation about the z axis (which is rotation in the (x,y) plane about its origin)

Cylinder coordinates in space are a combination of polar coords in a plane and the retained cartesian coord in an axis $\perp$ to the plane,

eg $x = r\cos\theta$ $\quad$ (x,y,z) cartes. coords
$y = r\sin\theta$ $\quad$ (r,θ,z) cylinder coords
$z = z$

They are particularly well suited in problems that have a rotational symmetry about an axis (the z axis)

(see first part of Ch. 12.7 for more examples)

Limits: (Sec 14.2)

The "informal" (hand-waving) definition of limits is as in SVCalc:

We say $\lim_{(x,y)\to(a,b)} f(x,y) = L$ if $f(x,y)$ gets arbitrarily close to L provided (x,y) is sufficiently close to (a,b).

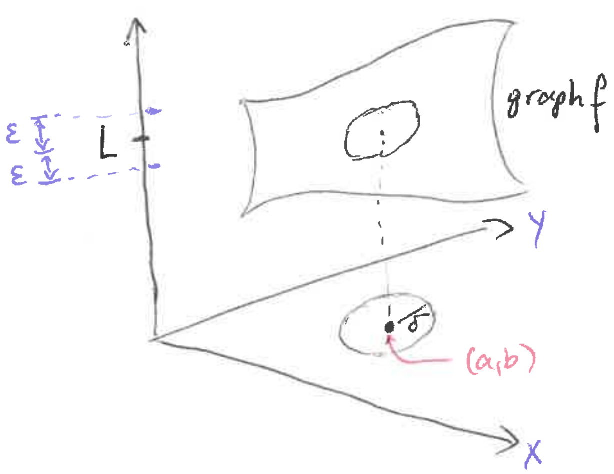
The formal mathematical definition translates this informal definition by clarifying the meanings of "arbitrarily close" and "sufficiently close". Lest the formal definition seem too abstract, let me explain that it can actually be understood in engineering terms:

"getting arbitrarily close" means that a "customer" may specify any error tolerance $\varepsilon > 0$ (in particular a very small one!), desiring that $|f(x,y) - L| < \varepsilon$. It must then be possible by choosing (x,y) sufficiently close to (a,b), i.e. by choosing a "design tolerance" δ, which means by requiring that $0 < \left\| \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} a \\ b \end{bmatrix} \right\| < \delta$, to guarantee that the output $f(x,y)$ of (x,y) meets the pre-specified error tolerance ε. Thus, we say

$$\lim_{(x,y)\to(a,b)} f(x,y) = L \quad \text{if}$$

for every $\varepsilon > 0$, there can be found a matching $\delta > 0$, such that, whenever $\underbrace{0 <}\left\| \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} a \\ b \end{bmatrix} \right\| < \delta$, then $|f(x,y) - L| < \varepsilon$

this little quirk is here for the purpose of allowing to discuss $\lim_{(x,y)\to(a,b)} f(x,y)$ even in situations where $f(a,b)$ is not defined.



~~Specify an arbitrar~~

Expect $\varepsilon > 0$ to be specified by a "customer" first.

Then you must be able to respond with a $\delta > 0$ (dependent on ε) that guarantees:

If $0 < \left\| \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} a \\ b \end{bmatrix} \right\| < \delta$, then

$$|f(x,y) - L| < \varepsilon$$

What we do not have in multi-variable, is a specified direction of approach (in SVC, you may have limits from the right and from the left, and there are no further choices).

Here, there are already infinitely many "straight-line" approaches

etc., but even these don't exhaust all possibilities, eg you could approach (a,b) on a "spiral": 

The definition of the limit merely refers to the distance btw (x,y) and (a,b), regardless of any directions

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|| We say f is continuous at (a,b), if $\lim\limits_{(x,y)\to(a,b)} f(x,y) = f(a,b)$ ||

The usual sum, product, quotient laws for limits carry over from SVC:

If $\lim\limits_{(x,y)\to(a,b)} f(x,y)$ and $\lim\limits_{(x,y)\to(a,b)} g(x,y)$ exist then

(a) $\lim\limits_{(x,y)\to(a,b)} \big(f(x,y)+g(x,y)\big) = \lim\limits_{(x,y)\to(a,b)} f(x,y) + \lim\limits_{(x,y)\to(a,b)} g(x,y)$

(a') same for $-$ $\qquad$ $-$

(b) same for $\cdot$ $\qquad$ $\cdot$

(c) same for $/$ $\qquad$ $/$

<u>provided</u> $\lim\limits_{(x,y)\to(a,b)} g(x,y) \neq 0$

Also, limits of components are easy

$$\lim_{(x,y)\to(a,b)} x = a \quad , \quad \lim_{(x,y)\to(a,b)} y = b$$

Analogs for more than two variables are obvious!

We also have the squeeze theorem:

If $g \leq f \leq h$ and $\lim g(x,y) = \lim h(x,y) = L$

then $\lim f(x,y) = L$ as well

[$(x,y) \to (a,b)$ beneath lim suppressed]

We get the usual rules about cont. fcts:

If f, g are cont. at (a,b) then $f+g$, $f-g$, $f \cdot g$ are cont. at (a,b), too

Likewise f/g, provided $g(a,b) \neq 0$.

Also: If g is cont. at (a,b) with $g(a,b) = c$

and f is cont. at c, then $f \circ g : (x,y) \mapsto f(g(x,y))$

is cont at (a,b)

Hwk: Read Examples #5, 6, 7 in Sec 14.2

(no grading hwk from this chapter)

Partial Derivatives

The spoiler first:

partial derivatives deserve their name, b/c they convey only partial information (not the total/complete info) and b/c they are not impartial, in the sense that they are "biased" in favor of coordinate directions.

I want to give you this ahead of time to counteract an impression to the contrary that you may otherwise absorb from the ease of calculation with them...

In a MV fct f, if you freeze all but one variable and study the SV fct of the remaining variable, then the SV derivative of this fct is called a partial derivative of f.