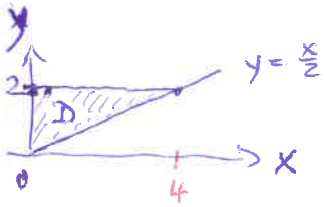
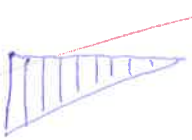
Crucial: Check out carefully all examples in Ch 15.2+3
(may skip "Excursion: Vol of Spheres in higher dim")

Example: $\iint_D e^{y^2}\,dA$ where D is



option 1: D is vertically simple



$$\int_0^4 \int_{x/2}^{2} e^{y^2}\,dy\,dx$$

Don't know an indefinite of e^{y^2} → stuck

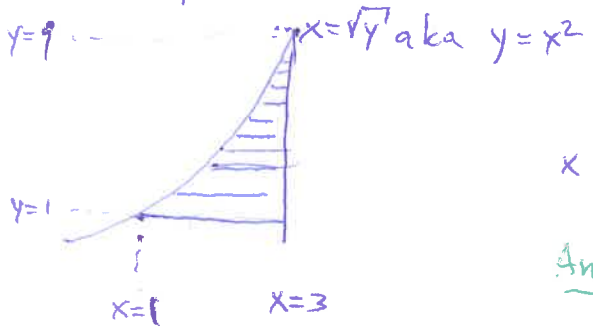
option 2: D is horizontally simple

$$\int_0^2 \int_0^{2y} e^{y^2}\,dx\,dy = \int_0^2 \left[e^{y^2}x\right]_{x=0}^{x=2y} dy$$

$$= \int_0^2 2y e^{y^2}\,dy = \left[e^{y^2}\right]_0^2 = \underline{e^4 - 1}$$

Example: Exchange the order of integration in

$$\int_1^9 \int_{\sqrt{y}}^{3} f(x,y)\,dx\,dy$$



x goes from $\sqrt{y}$ to 3

Answer:

$$\int_1^3 \int_1^{x^2} f(x,y)\,dy\,dx$$

Ex: Integrate $f(x,y,z) = $ ✗ over the region ~~bounded by~~ described as: $x \geq 0$, $y \geq 0$, $z \geq 0$

below ~~above~~ $z = 4 - x^2 - y^2$

above $z = x^2 + 3y^2$

Sol'n:

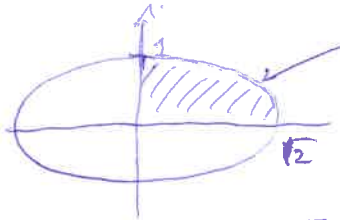
$$\iint_{\text{footprint of the region}} \int_{x^2+3y^2}^{4-x^2-y^2} \text{✗}\, dz\, dA$$

find the bdry of this region (in (x,y))

$4 - x^2 - y^2 = x^2 + 3y^2$ (that's where lower & upper surface intersect)

$2x^2 + 4y^2 = 4$

$x = \sqrt{2 - 2y^2}$ or $y = \sqrt{1 - \frac{1}{2}x^2}$



$$\int_0^1 \int_0^{\sqrt{2-2y^2}} (\cdots)\, dx\, dy \quad \text{or} \quad \int_0^{\sqrt{2}} \int_0^{\sqrt{1-\frac{1}{2}x^2}} (\cdots)\, dy\, dx$$