Example for using polar coordinates:

Cardiod can be written as $R(\theta) = a(1-\cos\theta)$

(That's NOT how you've seen it – we had done it as a param. curve)
Trust me that the two ways describe the same cardiod.

Question: What is the area enclosed?

D: $0 \le r \le R(\theta)$
$0 \le \theta \le 2\pi$

$$\text{area}(D) = \iint_D 1\, dA = \int_0^{2\pi} \underbrace{\int_0^{R(\theta)} 1 \cdot r\, dr}_{\left[\frac{1}{2} r^2\right]_0^{R(\theta)}}\, d\theta = \frac{1}{2}\int_0^{2\pi} R(\theta)^2\, d\theta = \ldots$$

$$d\text{area} = \frac{1}{2} R^2\, d\theta$$

You probably have seen this formula in SVC (calc 2)

$$\ldots = \frac{1}{2} a^2 \int_0^{2\pi} (1-\cos\theta)^2\, d\theta = \frac{1}{2} a^2 \int_0^{2\pi} \left(1 - 2\cos\theta + \cos^2\theta\right) d\theta$$

$$= [\ldots] = \frac{1}{2} a^2 \left(2\pi - 0 + \pi\right)$$

$$= \frac{3\pi}{2} a^2$$

Cylindrical coordinates:

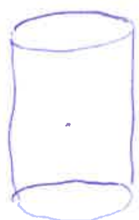
$x = r\cos\theta$
$y = r\sin\theta$
$z = z$

$$dV = r\, dr\, d\theta\, dz$$

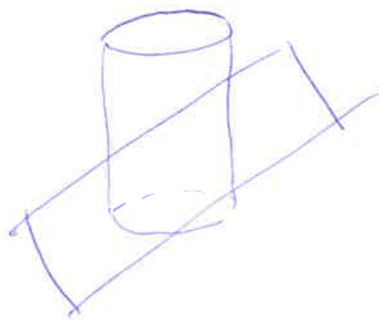
base is a curvilinear rectangle as in polar coord's

May use cylindrical coords if your domain W can be conveniently described by $z_1(r,\theta) \le z \le z_2(r,\theta)$ $r_1(\theta) \le r \le r_2(\theta)$ $\theta_2 \le \theta \le \theta_1$

$$\iiint_W f(x,y,z)\,dV = \int_{\theta_1}^{\theta_2}\int_{r_1(\theta)}^{r_2(\theta)}\int_{z_1(r,\theta)}^{z_2(r,\theta)} f(r\cos\theta, r\sin\theta, z)\,r\,dz\,dr\,d\theta$$



a cylinder with z-axis as its axis is the typical paradigm



piece of cyl. cut off with some plane

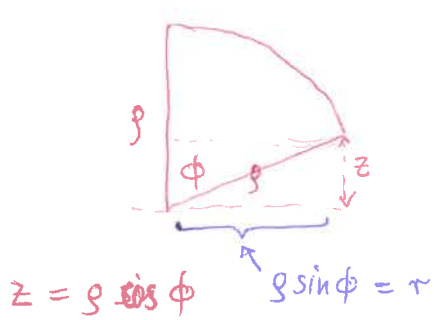
Spherical coordinates (cf. Sec 12.7)

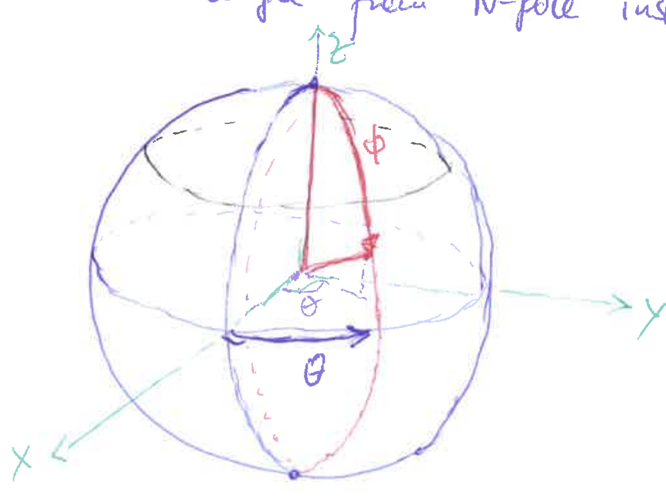
(useful for spherical symmetry, eg W = ball)

ρ distance from origin

two angles (eg. geographical longitude and latitude)

physicists' variant uses angle from N-pole instead of latitude





$$\boxed{\begin{aligned} x &= \rho \sin\phi \cos\theta \\ y &= \rho \sin\phi \sin\theta \\ z &= \rho \cos\phi \end{aligned}}$$

$$\sqrt{x^2+y^2+z^2} = \rho$$

$$\tan\theta = \frac{y}{x} \qquad \text{or} \quad \cot\theta = \frac{x}{y}$$

$$\tan\phi = \frac{\sqrt{x^2+y^2}}{z}$$

The little "boxes" in the Riemann sum would be

base:

parallel of latitude

parallels of latitude

parallels of longitude

their area: $r\,d\theta$ width where $r = \rho\sin\phi$

$\rho\,d\phi$ height

$$\rho^2 \sin\phi \, d\theta \, d\phi$$

"thickness" $d\rho$

$$dV = \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$