No notes available from Oct 22 + Oct 24

refer to chapters 15.6 and 15.5

instead

Applications:

(1) Density δ (of some mixed material so density may depend on location

$$\iiint_W \delta(x,y,z)\, dV = \text{mass}(W)$$

(2) center of mass:

- in case of constant density, you can view the COM as the average of the position vectors of all points in W

$$x_{CM} = \iiint_W x \, dV \Big/ \iiint_W dV$$

$$y_{CM} = \iiint_W y \, dV \Big/ \quad \text{-''-}$$

$$z_{CM} = \iiint_W z \, dV \Big/ \quad \text{-''-}$$

- in case of varying density δ

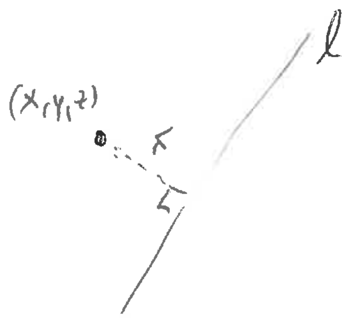
$$x_{CM} = \iiint_W x \, \delta(x,y,z)\, dV \Big/ \iiint_W \delta(x,y,z)\, dV$$

$$y_{CM} = \iiint_W y \, \delta(x,y,z)\, dV \Big/ \quad \text{-''-}$$

$$z_{CM} = \iiint_W z \, \delta(x,y,z)\, dV \Big/ \quad \text{-''-}$$

Moment of inertia (for rotation about an axis l)

Let r be the distance of a point (x,y,z) from l



Then the moment of inertia of a body W with density δ about l will be

$$\iiint_W r(x,y,z)^2 \, \delta(x,y,z)\, dV$$

dV with mass $\underbrace{\delta(x,y,z)\,dV}_{dm}$

When calc'ing the moment of inertia of a point (little lump) w mass dm we're calc $r^2\,\delta m$

Typically we study the M.o.I. for l one of the coordinate axes:

$$I_z = \iiint_W (x^2+y^2)\ \delta(x,y,z)\, dV$$

(where l is the z-axis)

$$I_x = \text{—''—} \quad (y^2+z^2) \quad \text{—''—}$$

$$I_y = \text{—''—} \quad (x^2+z^2) \quad \text{—''—}$$

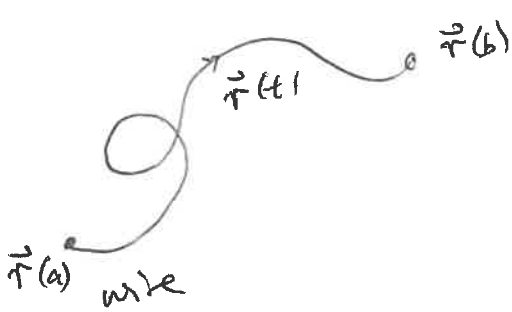
if density is const, can pull it out and eg $\iiint(x^2+y^2)\,dV$ etc

Line Integrals:

These are integrals where the domain integration is a curve (in the plane, or in space)

The "differential" will then of course NOT be dA or dV but ds (a little bit of length)

Eg: COM of a thin wire



Along this curve we have a parameter t

curve described as $\vec{r} = \vec{r}(t)$

$a \leq t \leq b$

On this curve (at least there) there is a function f defined

f may or may not be defined in the ambient space around the curve

f could be eg $\underbrace{\text{mass density}}_{dm/ds}$ of the wire ~~along~~

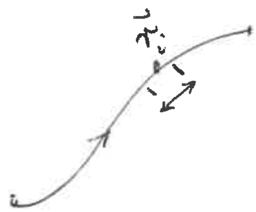
then: total mass $\displaystyle\int_{\text{wire}} f(\vec{r})\, ds$ this is an example of a (scalar) <u>line integral</u>.

Def: Let C be a parametrized curve $\vec{r} = \vec{r}(t)$
$a \leq t \leq b$.

and f a function def'd (at least) on this curve

Then $$\int_C f(\vec{r})\, ds := \int_a^b f(\vec{r}(t)) \|\vec{r}\,'(t)\| dt \qquad (*)$$

→ eval is a Calc 2 project



Riem. sum $\sum_i f(\vec{r}_i) \cdot \Delta s_i =$

$\vec{r}_i = \vec{r}(t_i)$

$\Delta s_i \approx \|\vec{r}\,'(t_i)\| \Delta t_i$

$(*)$ is called a (scalar) line integral.

BTW the line integral depends on f, C, but NOT on the chosen parametrization (as long as the orientation of C is maintained). Consequence of u-sub rule in SVC;

Suppose you have a parametrization by t,
your neighbor parametrizes by τ, and $t = t(\tau)$

$$\int_{t(\alpha)}^{t(\beta)} f(\vec{r}(t))\,\|\vec{r}\,'(t)\|\,dt \overset{?}{=} \int_{\alpha}^{\beta} f(\vec{r}(t(\tau)))\,\|\underbrace{\dot{\vec{r}}(\tau)}\|\,d\tau$$

$$\frac{d\vec{r}}{d\tau} = \frac{d\vec{r}}{dt}\,\frac{dt}{d\tau}$$

$t(\tau)$ = new variable t

$$\int_{t(\alpha)}^{t(\beta)} f(\vec{r}(t))\,\|\vec{r}\,'(t)\| \cdot \underbrace{\underbrace{\left|\frac{dt}{d\tau}\right|}_{=\frac{dt}{d\tau}\ \text{b/c}\ \frac{dt}{d\tau}>0}\,d\tau}_{=dt}$$