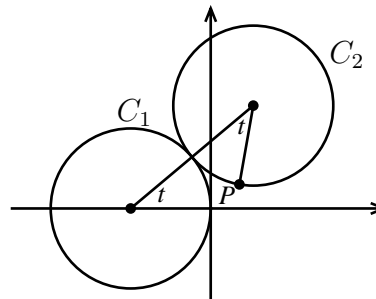


The Cardioid:

A circle C_1 of radius 1 sits stationary with center $(-1, 0)$. Another circle C_2 of radius 1 touches it from the right, in the origin. This circle is soon to roll along the fixed circle C_1 without sliding. A point P is marked on the circle C_2 . Initially it is the point where both circles touch. As C_2 rolls along C_1 , the point P traces out a curve in the stationary plane. Use t for the angle (measured on C_1) of the point of contact of both circles.



(a) Give a vector valued function $t \mapsto \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ that describes the position vector of P as a function of t .

The curve traced out by P is called a cardioid. Its name comes from the Greek word for 'heart', and you'll see why when you graph this curve.

(b) Calculate the length of the cardioid. (Geometry will determine the limits of integration.)