

## Some Tongue-in-Cheek Material to Explain why we Need a Language Course

A student of biology, a student of physics, and a student of mathematics go to Scotland for a holiday. Driving in the countryside, they see a black sheep standing on the meadow. The biology student is surprised: "I didn't know that the sheep in Scotland are black!" — The physics student doesn't agree: "You really cannot conclude from observing a single black sheep that all sheep in this country are black. All you can infer is that there are some black among the sheep in Scotland" — The math student is still not content: "You physics guys aren't particularly precise either! As a matter of fact, all you can conclude is that there is at least one sheep in Scotland that appears to be black at least on one side!"

Well, the others have the laughter on their side about how clumsy we mathematicians are sometimes. But there is good reason for this precision. In his introductory paragraph to his book "A radical approach to real analysis", the author David Bressoud points out that the invention of Fourier series in the early 1800's caused a crisis in mathematics that shook the edifice of calculus so thoroughly that it had to undergo a 50 year period of reconstruction from the foundations to the pinnacle. At that time, all the achievements you learnt in 1st year Calculus had been accomplished already, and a lot more. Nevertheless, when pursuing things further and deeper, mathematicians realized that things didn't fit together anymore and that concepts like "convergence" were not as clear to them as they had thought. We are not going the historic way here, and we hardly do anything beyond precalculus in this course. But the precision and the language that is needed for it did arise from such foundational crises, and so it is for good reason that you learn this language.

I want to show you a few odd observations from the study of language to convince you that there are ambiguities in everyday language that we cannot tolerate for mathematical reasoning: Compare the following three English sentences:

Don't drink and drive

Don't kill and steal

Don't grumble and eat your lunch

Even though they are construed analogously, they have a *structurally* very different meaning each. Do you see it? How about the following?

President Jefferson initiated the Lewis and Clarke expedition

can be rephrased in passive voice:

The Lewis and Clarke expedition was initiated by President Jefferson

Let's try the rephrasing in passive voice again:

Beavers build dams

can be rephrased in passive voice (can't it?):

Dams are built by beavers

Well, we'll soon encounter a mathematical concept that is not expressed, but only implied, within these sentences and that will explain the difference.

We also insist on precise definitions of concepts, even when they sometimes may seem obvious. If definitions get metaphoric or wavy, we can draw funny conclusions:

John Doe is a real skunk

Skunks have black fur with white stripes

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Conclusion: John Doe has black fur with white stripes

In practical situations, where we have world knowledge about sheep, presidents, dams, beavers and skunks, about lunch and DUI, we clarify ambiguities automatically without even discussing them. When we venture into unexplored territory with no world knowledge about the things we are going to study, ambiguities won't disappear automatically. To Bertrand Russell the following quote is attributed:

Mathematics is the subject in which we never know what we are talking about, nor whether what we are saying is true

Maybe you got this impression sometimes in earlier classes. Then it's about time that you are taking M300. Then you can deal with not knowing (not having a-priori world knowledge of) the things we are talking about, and you will know how to figure out what is true. (I am recklessly distorting the context of the 2nd part of Russell's claim, which actually refers to more subtle foundational questions not discussed in this course.)