M351

#3: ((Closure properties trivial))

(C+) : $(A+B)_{ij} \underset{\substack{\uparrow \\ \text{Def of } +}}{=} A_{ij} + B_{ij} \underset{\substack{\uparrow \\ \text{(C+) in } R}}{=} B_{ij} + A_{ij} \underset{\substack{\uparrow \\ \text{Def of } +}}{=} (B+A)_{ij}$

(A+) analogously

(N+) The matrix consisting of all 0's is the neutral for addition

(Inv+) The ~~negative~~ add'inv. of A is the matrix with entries

$-A_{ij}$

(N·) Let I be the matrix with $I_{ij} = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{else} \end{cases}$

(if R has an identity 1)

Claim $A \cdot I = A$

$(A\cdot I)_{ik} = \sum_j A_{ij} I_{jk} = A_{ik}$ since only the term $j=k$ ~~so~~ remains in the sum

~~(A·)~~ ($I \cdot A = A$ sim'ly)

(A·) $((A\cdot B)\cdot C)_{il} = \sum_k (A\cdot B)_{ik} C_{kl} = \sum_k \left(\left(\sum_j A_{ij} B_{jk} \right) C_{kl} \right) =$

$\underset{\substack{\uparrow \\ (D)^* \text{ in ring } R}}{=} \sum_k \sum_j \left((A_{ij} B_{jk}) C_{kl} \right) \underset{\substack{\uparrow \\ (A\cdot) \text{ in } R}}{=} \sum_k \sum_j \left(A_{ij} (B_{jk} C_{kl}) \right) =$

$\underset{\substack{\uparrow \\ (C+)^* \text{ in } R}}{=} \sum_j \sum_k A_{ij} (B_{jk} C_{kl}) \underset{\substack{\uparrow \\ (D)^* \text{ in } R}}{=} \sum_j \left(A_{ij} \left(\sum_k B_{jk} C_{kl} \right) \right) =$

$= \sum_j A_{ij} (BC)_{jl} = (A(BC))_{il}$

Hence $(AB)C = A(BC)$