Rmks: • The starred axioms like $(D)^*$ actually include an induction proof: (D) guarantees, e.g., $a(b_1+b_2) = ab_1 + ab_2$ from which the version with n terms

$$a(b_1+b_2+\ldots+b_n) = ab_1+ab_2+\ldots+ab_n$$

follows by induction

• Moreover, in using the Σ notation, we have already tacitly absorbed (A+) since Σ doesn't specify parentheses.

• Less hindsight in naming the summation indices may require renaming them later.

(D) $$\big(A(B+C)\big)_{ik} = \sum_j A_{ij}\,(B+C)_{jk} \underset{\text{(D) in } \mathbb{R}}{=} \sum_j \big(A_{ij}B_{jk} + A_{ij}C_{jk}\big)$$

$$\underset{\text{(C+) in } \mathbb{R}}{=} \sum_{j} A_{ij}B_{jk} + \sum_j A_{ij}C_{jk} = (AB)_{ik} + (AC)_{ik} = (AB+AC)_{ik}$$

$(B+C)A = BA + CA$ analogously.

Note: I have generally written $\sum_j$ as shorthand for $\sum_{j=1}^{n}$, since the range of the indices is clear.

#4: (C+), (A+), (N+), (Inv+) are the same as in the direct sum and straightforward; $(0,0)$ is the neutral for addition

(D) $$\underbrace{\big((x_1,y_1)+(x_2,y_2)\big)\cdot(u,v)} \overset{?}{=} \underbrace{(x_1,y_1)\cdot(u,v) + (x_2,y_2)\cdot(u,v)}$$

$$= (x_1+x_2,\, y_1+y_2)\cdot(u,v) =$$
$$= \big((x_1+x_2)u - (y_1+y_2)v,\; (x_1+x_2)v + (y_1+y_2)u\big)$$

$$= (x_1u - y_1v,\, x_1v+y_1u) + $$
$$+ (x_2u - y_2v,\, x_2v + y_2u)$$
$$= \big((x_1u-y_1v)+(x_2u-y_2v),\; (x_1v+y_1u)+(x_2v+y_2u)\big)$$