Comparing components:

$$(x_1+x_2)u-(y_1+y_2)v \underset{\uparrow}{=} x_1u+x_2u-y_1v-y_2v =$$

(D) in R
(A+) by omission of (...)
properties of (Inv+) : Book p16 Prop4.1

$$\underset{\uparrow\,(C+)}{=} x_1u-y_1v+x_2u-y_2v = (x_1u-y_1v)+(x_2y-y_2v)$$ as on righthand side

Similarly for the 2^{d} component

(A$\cdot$) $\big((x,y)(u,v)\big)(a,b) = (xu-yv,\ xv+yu)(a,b) =$

$$= \big((xu-yv)a-(xv+yu)b,\ (xu-yv)b+(xv+yu)a\big)$$

$(x,y)\big((u,v)(a,b)\big) = (x,y)\,(ua-vb,\ ub+va) =$

$$= \big(x(ua-vb)-y(ub+va),\ x(ub+va)+y(ua-vb)\big)$$

Both sides equal

$$(xua-yva-xvb-yub,\ xub-yvb+xva+yua)$$

This uses (A$\cdot$), (A+), (C+), and Prop4.1 as before, but <u>never</u> (C$\cdot$)
(and it <u>mustn't</u>!)

#5: • If 1 is identity in R, then (1,0) is identity in R[i]

Pf: $(x,y)(1,0) = (x\cdot 1-y\cdot 0,\ x\cdot 0+y\cdot 1) = (x,y)$ (using $a\cdot 0=0$ ($\forall a$))

$(1,0)(x,y) = (1\cdot x-0\cdot y,\ 1\cdot y+0\cdot x) = (x,y$

• If R is commutative, then

$$(x,y)(u,v) = (xu-yv,\ xv+yu) \underset{\substack{\uparrow \\ (C\cdot)\text{ in } R}}{=} (ux-vy,\ vx+uy)$$

$$(u,v)(x,y) = (ux-vy,\ uy+vx)$$

= (using (C+))