

Since $b \neq 0$, b has an inverse b^{-1} .

$$a^2 + b^2 = 0 \Rightarrow \underbrace{a^2 b^{-1} \cdot b^{-1}}_{(ab^{-1})(ab^{-1})} + \underbrace{b \cdot b \cdot b^{-1} \cdot b^{-1}}_1 = 0$$

So (2) fails with $x = ab^{-1}$.

#6: $i := (0, 1)$; then $i \cdot i = (0, 1) \cdot (0, 1) = (0 \cdot 0 - 1 \cdot 1, 0 \cdot 1 + 1 \cdot 0) = (-1, 0)$

so $i^2 = -1$ (the negative of the mult. identity)

$\mathbb{R}[i]$ is known as $\mathbb{C}$

$(a, 0)$ is usually called a which makes sense, since all operations with $(*, 0)$ work just as in $\mathbb{R}$ (we'll get this more formally later)

$(0, 1)$

i

$(0, b)$

bi

(a, b)

$a + bi$

The given rules are just the usual rules for multiplying complex numbers.

$\mathbb{R}[i]$ is a field since $x^2 + 1 \neq 0$ in $\mathbb{R}$ (see #5, end)

The calc' of the inverse ^{in #5} ~~becomes~~ will look a bit more familiar if you know how to take reciprocals of complex numbers.

#7: One way to do it is to check all eight cases for an element

x : ~~in A~~ $\in A$ or $\notin A$
 $\in B$ or $\notin B$
 $\in C$ or $\notin C$