#8: <u>Step 1</u> $\underbrace{(b+b)(b+b)}_{\substack{=b+b \\ \text{by the } a^2=a \text{ property}}} \underset{\substack{\uparrow \\ \text{(D) used twice} \\ (A+)}}{=} bb+bb+bb+bb \underset{\substack{\uparrow \\ b^2=b \text{ extra property} \ \& \ (A+)}}{=} (b+b)+(b+b)$

So we have $\quad b+b = (b+b)+(b+b)$

hence $\quad 0 = b+b \quad$ (by adding $-(b+b)$)

<u>Step 2</u>: Note that the previous property actually makes + and − the same thing: $b=-b$

So I could have started with $(bc+cb)(bc+cb)$ instead.

But let's stick with what we have:

$(bc-cb)(bc-cb) = bc-cb \qquad$ by the $a^2=a$ property

$(bc-cb)(bc-cb) \underset{\substack{\uparrow \\ \text{(D) twice} \\ (A+), \text{ Prop 4.1.} \\ (A\cdot)}}{=} (bc)(bc) - c(bb)c - b(cc)b + (cb)(cb) =$

$= bc - cbc - bcb + cb$

Comparing we get $\quad bc-cb = bc-cbc-bcb+cb$

(adding $bc+cb$, using Step 1 to get $bc+bc=0$) $\quad \Rightarrow \quad 0 = -cbc-bcb$

$\Rightarrow \quad bcb = -cbc$

$\Rightarrow \quad bcb = cbc \qquad$ (again with step 1)

<u>Step 3</u>: Multiply $bcb = cbc$ by b — from the right / from the left

get — $bcbb = cbcb \quad \Rightarrow \quad bcb = cb$
$\quad\;\; bbcb = bcbc \quad \Rightarrow \quad bcb = bc$ } Therefore $\underline{cb=bc}$