This counts each pair twice once as (A,B), once as (B,A)

So we have $\frac{1}{2}(4\cdot 7 + 6\cdot 3 + 4\cdot 1) = \underline{\underline{25}}$ pairs

NOT PART OF HWK

[The general formula for a set of m elements is:

$$\frac{1}{2}\sum_{k=1}^{m}\binom{m}{k}(2^{m-k}-1) = \frac{1}{2}\left(\sum_{k=1}^{m}\binom{m}{k}2^{m-k}\cdot 1^k - \sum_{k=1}^{m}\binom{m}{k}\right)$$

$k = \#$ elements in A

$$= \frac{1}{2}\left((3^m - 2^m) - (2^m - 1)\right)$$

$$= \underline{\underline{\frac{1}{2}(3^m+1) - 2^m}}$$

#11: If $1=0$, then $a\cdot 1 = a\cdot 0$ for every $a \in R$

$a\cdot 1 = a$ by (N_0)

$a\cdot 0 = 0$ by ring axioms

So we conclude $a = 0$ for every $a \in R$,

i.e. R consists only of the zero element.

#12: $b^{-1}a^{-1}$ is an inverse of ab (and therefore by uniqueness the inverse)

Proof: We must show $(b^{-1}a^{-1})(ab) = 1$

and $(ab)(b^{-1}a^{-1}) = 1$

$$(b^{-1}a^{-1})(ab) = ((b^{-1}a^{-1})a)b = (b^{-1}(a^{-1}a))b = (b^{-1}1)b = b^{-1}b = 1$$

$$(ab)(b^{-1}a^{-1}) = ((ab)b^{-1})a^{-1} = (a(bb^{-1}))a^{-1} = (a\,1)a^{-1} = aa^{-1} = 1$$