Alternative sol'n for #7

(1) Want to show $(A+B)+C \subseteq A+(B+C)$

Suppose $x \in (A+B)+C$.

$$= \big((A+B) \setminus C\big) \cup \big(C \setminus (A+B)\big)$$

Case 1: $x \in A+B$, but $x \notin C$. Note that $A+B = (A \setminus B) \cup (B \setminus A)$

Subcase 1a: $x \in A$, $x \notin B$; $x \notin C$

Since $x \notin B$, we get $x \notin (B \setminus C)$

Since $x \notin C$, -"- $x \notin (C \setminus B)$

Therefore $x \notin (B \setminus C) \cup (C \setminus B) = (B+C)$

But $x \in A$, so $x \in A \setminus (B+C)$

and therefore $x \in A+(B+C)$

Subcase 1b: $x \notin A$, $x \in B$; $x \notin C$

~~Since $x \notin A$, we get $x \notin A \setminus (B+C)$~~

Since $x \in B$ but $x \notin C$, we get $x \in (B+C)$

Furthermore, since $x \notin A$, we find $x \in (B+C) \setminus A$

Therefore $x \in A+(B+C)$

Case 2: $x \notin A+B$, but $x \in C$

Note that $x \notin A+B$ means: $x \in A \cap B$ or x is neither in A nor in B

Subcase 2a: $x \in A \cap B$, $x \in C$

Since $x \in B$ and $x \in C$, we get $x \notin B+C$

But $x \in A$; so $x \in A \setminus (B+C)$

Hence $x \in A+(B+C)$

Subcase 2b: $x \notin A$, $x \notin B$, $x \in C$

Since $x \notin B$, but $x \in C$, we get $x \in (B+C)$.

Since also $x \notin A$, we get $x \in (B+C) \setminus A$

Hence $x \in A+(B+C)$