#13: The properties (C+), (A+), (A·), (D) are inherited from $C^0[0,1]$. (so is (C·))

- We need to check <u>closure</u> and (N+), (Inv+)
- <u>Closure</u>

  If $f, g \in C^0_A[0,1]$, we must show that $f+g, f\cdot g \in C^0_A[0,1]$

  ~~[illegible]~~: Clearly $f+g, f\cdot g \in C^0[0,1]$ (i.e. are cont. fcts).

  Remains to show: $f(x)+g(x)=0$, $f(x)g(x)=0$ <u>for all $x \in A$</u>

  This is true since $f(x)=g(x)=0$ for all <u>$x \in A$</u>

- <u>(N+)</u> The zero function ($f(x)=0$ for all $x \in [0,1]$) does lie in $C^0_A[0,1]$ and serves as zero element there.

- <u>(Inv+)</u> If $f(x)=0$ for all $x \in A$, then $-f(x)=0$ for all $x \in A$.

  So ~~the~~ $-f$ serves as a (the) additive inverse of f <u>within</u> $C^0_A[0,1]$

<u>Note:</u> (not req'd) With some trivial exceptions, for a typical set A, we do lose the identity $\in C^0[0,1]$. To avoid technicalities that you could only understand with M341 knowlege, let's assume A is a closed interval, say $[\frac{1}{3}, \frac{2}{3}]$ strictly contained in $[0,1]$.

For f to be the identity in $C^0_A[0,1]$, we would need

$f(x)g(x)=g(x)$ for <u>all</u> $g \in C^0_A[0,1]$ and all $x \in [0,1]$.

Since for any $x_0 \notin A$, there is some function $g \in C^0_A[0,1]$ that does not vanish at x_0 ($g(x_0) \neq 0$), we conclude $f(x_0)=1$ for those x_0

But $f(x)=0$ for $x \in A$. A continuous fct however cannot be 0 on A and 1 outside A (by the intermediate value theorem).