#14a: $\mathbb{Z} \oplus \{0\}$ is a subset of $\mathbb{Z} \oplus \mathbb{Z}$, and is a ring in its own right, with the <u>same</u> operations $+$ and $\cdot$ as are defined on $\mathbb{Z} \oplus \mathbb{Z}$.

So this makes $\mathbb{Z} \oplus \{0\}$ clearly a subring.

- The ~~unity (aka~~ identity} in $\mathbb{Z} \oplus \mathbb{Z}$ is $(1,1)$
- $(1,1) \notin \mathbb{Z} \oplus \{0\}$
- $(1,0)$ is an (the) identity in $\mathbb{Z} \oplus \{0\}$, (but not in $\mathbb{Z} \oplus \mathbb{Z}$)

#14b: Again $\mathcal{P}(N)$ is a subset of $\mathcal{P}(M)$ ~~with~~ and is a ring in its own right, with the same operations.

- The identity in $\mathcal{P}(M)$ is M
- $M \notin \mathcal{P}(N)$
- N is an (the) identity in $\mathcal{P}(N)$

#15: Suppose R is a ring, S a subring of R

Let's assume 0_R is the additive identity in R

0_S -"- -"- in S

Then $0_S + 0_S = 0_S$. This is an eqn of elements of R (as well as one of elements in S). We add the additive inverse (in R) of 0_S to both sides.

This gives 0_R on the rhs and $0_S + 0_R = 0_S$ on the lhs

So we obtain $0_S = 0_R$

In short, the existence of an inverse (and the cancellation laws implied from it) ensure that $0_S = 0_R$