#16 cont'd: (typo: read $p_7 = x^2 + 25x^{11}$ instead of $p_7 = x^2 + 25x^{1}1$)

$p_9 < p_{10} < p_5 < p_2 < p_7 < p_3 < p_6 < p_8 < p_4 < p_1$

#17: (Proof of order properties wasn't required here, because it would be done in perfect analogy to the one in #16)

Ordering:

$p_8 < p_3 < p_4 < p_1 < p_2 < p_6 < p_5 < p_9 < p_{10} < p_7$

#18: Proof by induction: for $n=1$, we merely need to show: $(ab)^1 = a^1 b^1$. Since $x^1 = x$ for any x, this is true:

$$ab = ab$$

Induction step $n \to n+1$

$$(ab)^{n+1} \underset{\substack{\text{by recursive def'n} \\ \text{of power}}}{=} (ab)^n (ab) \underset{\substack{\text{by induction} \\ \text{hypothesis}}}{=} (a^n b^n)(ab) \underset{\substack{\text{by (C}\cdot\text{)} \\ \text{and (A}\cdot\text{)}}}{=} a^n((b^n b) a) = (a^n a)(b^n b)$$

$$= a^{n+1} b^{n+1}$$

#19: For $n=1$, we have the triviality $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

Induction step $n \to n+1$:

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{n+1} \underset{\substack{\text{by recursive def'n of} \\ \text{power}}}{=} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{n} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \underset{\text{by ind. hypothesis}}{=} \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \underset{\substack{\text{rules of matrix} \\ \text{multiplication}}}{=} \begin{bmatrix} 1 & n+1 \\ 0 & 1 \end{bmatrix} \quad \text{qed.}$$