(39 a) $(1,1,1,1,1,\dots)\cdot(0,1,2,3,4,5,\dots) =$

$(1\cdot 0,\ 1\cdot 1+1\cdot 0,\ 1\cdot 2+1\cdot 1+1\cdot 0,\ 1\cdot 3+1\cdot 2+1\cdot 1+1\cdot 0,\ \dots)$

$= (0, 1, 2+1, 3+2+1, \dots) = (0, 1, 3, 6, 10, 15 \dots)$

The entry $(ab)_{30}$ is $29+28+\dots+2+1+0 = \frac{1}{2}\cdot 29\cdot 30 = \underline{\underline{435}}$

39(b): The sum being defined as $(a+b)_n := a_n + b_n$,
the addition axiom are proved as they are proved for the direct sum of rings, namely:

(C+) $a+b = b+a$ because for all n

$$(a+b)_n = a_n + b_n \underset{\text{(C+) in } R}{=} b_n + a_n = (b+a)_n$$

(A+) $$((a+b)+c)_n = (a+b)_n + c_n = (a_n+b_n)+c_n \underset{\text{(A+) in } R}{=} a_n + (b_n + c_n)$$
$$= a_n + (b+c)_n = (a+(b+c))_n$$

(N+) The sequence $(0,0,0,\dots)$ consisting of ~~all~~ zeros only is the additive identity

(Inv+) The negative of $(a_0, a_1, a_2, a_3, \dots)$ is $(-a_0, -a_1, -a_2, -a_3 \dots)$

(C·) $$(ab)_n = \sum_{i=0}^{n} a_i\, b_{n-i} \qquad (ba)_n = \sum_{j=0}^{n} b_j\, a_{n-j}$$

We must show that these two are equal.

$$\sum_{i=0}^{n} a_i\, b_{n-i} \underset{\text{(C·) in } R}{=} \sum_{i=0}^{n} b_{n-i}\, a_i = \sum_{j=0}^{n} b_j\, a_{n-j}$$

new index of summation $j = n-i$ uses (C+) and (A+) in R.