#41: We need to show closure properties wrt. additive inverse, addition & multiplication. Only the latter isn't quite trivial: If $a_n = 0$ for $n \geq n_0$

and $b_n = 0$ for $n \geq n_1$

then $(ab)_n = 0$ for $n \geq n_0 + n_1$

$$(ab)_n = \sum_{i=0}^{n} a_i \, b_{n-i}$$

$\hookrightarrow 0$ if $i \geq n_0$ $\quad$ 0 if $n-i \geq n_1$

So terms only contribute if $i < n_0$ **and** $n-i < n_1$

which couldn't happen unless $n < n_0 + n_1$

(That's good enough for closure. With more care, we could say: ~~...~~ ... only contribute if $i \leq n_0 - 1$, $n-i \leq n_1 - 1$

which couldn't happen unless $n \leq n_0 + n_1 - 2$)

$(1, 2, 0, -7, 3) \cdot (2, -1, 4) =$

$(2, 3, 2, -6, 13, -31, 12)$

#42: X^0 is the mult. identity (1)

$X^1 = (0, 1)$

$X^2 = (0, 0, 1)$

$X^3 = (0, 0, 0, 1)$ etc.

$(1, 2, 0, -7, 3) = 1 \cdot X^0 + 2 \cdot X^1 + 0 \cdot X^2 - 7X^3 + 3X^4$

This identifies the elements of $\mathbb{Z}[X]$ as polynomials and also sheds light on the meaning of the "funny-looking" Definition of multiplication