

#44: $p = 2X+1$, $q = 3X+1$ both have degree 1

$$pq = 6X^2 + 5X + 1 = 5X + 1 \text{ has degree } 1 < 2$$

We can even make up a more radical example:

$$p = 2X^n \quad q = 3X^m \quad pq = 0$$

#45: If $\deg r < \deg b$, and $a = bq + r$, then

$$\deg a = \deg b + \deg q$$

($\mathbb{Z}$ is an integral domain, so we may use the degree formula.)

So if we could find q, r as req'd then $\deg q$ would have to be 1. But then $q = q_1X + q_0$

$$a - bq = \frac{1}{2} (1 - 2q_1)X^3 + (1 - 2q_0)X^2 + 2X + 1$$

For this to have degree < 2 , we would need

$$1 - 2q_1 = 0 \quad (\text{and also } 1 - 2q_0 = 0, \text{ which we won't use})$$

But $1 - 2q_1 = 0$ is impossible for $q_1 \in \mathbb{Z}$