#53: Whenever $\mathbb{Z}_p$ has an ~~number~~ element i st. $i^2+1=0$
the factorization $(X^2+i)(X^2-i)$ is possible.

Whenever $\mathbb{Z}_p$ has an element z st. $z^2-2=0$
the factorization $(X^2+zX+1)(X^2-zX+1)$ is possible.

Whenever $\mathbb{Z}_p$ has an element u st. $u^2+2=0$
the factorization $(X^2+uX-1)(X^2-uX-1)$ is possible.

If two of the above hold (eg if we have elements i, z)
then they hold all three (namely in this example $u=iz$
satisfies $u^2=i^2z^2=(-1)(2)=-2$; and
similarly in the other cases), and then we have the
factorization $\left(X-2^{-1}(iz+z)\right)\left(X-2^{-1}(iz-z)\right)\left(X-2^{-1}(-iz+z)\right)\left(X-2^{-1}(-iz-z)\right)$
This latter case does not ~~happen~~ apply for $p=2$ (where 2^{-1} doesn't exist).

Factorizations of X^4+1 are:

$(X+1)^4$ in $\mathbb{Z}_2[X]$

$(X^2+X-1)(X^2-X-1)$ in $\mathbb{Z}_3[X]$

$(X^2+2)(X^2-2)$ in $\mathbb{Z}_5[X]$

$(X^2-3X+1)(X^2+3X+1)$ in $\mathbb{Z}_7[X]$

$(X^2+3X-1)(X^2-3X-1)$ in $\mathbb{Z}_{11}[X]$

$(X^2+5)(X^2-5)$ in $\mathbb{Z}_{13}[X]$

$(X-2)(X+2)(X-8)(X+8)$ in $\mathbb{Z}_{17}[X]$

$(X^2-6X-1)(X^2+6X-1)$ in $\mathbb{Z}_{19}[X]$

$(X^2-5X+1)(X^2+5X+1)$ in $\mathbb{Z}_{23}[X]$

$(X^2+12)(X^2-12)$ in $\mathbb{Z}_{29}[X]$

p	$x^2+1=0$	$x^2-2=0$	$x^2+2=0$
2	1	0	0
3	DNE	DNE	1
5	2	DNE	DNE
7	DNE	3	DNE
11	DNE	DNE	3
13	5	DNE	DNE
17	4	6	7
19	DNE	DNE	~~4~~6
23	DNE	5	DNE
29	12	DNE	DNE