Hwk for M443

Hwk 19: Calculate $\lim_{x\to 0} \frac{\tan x - \sin x - \frac{1}{2}x^3}{x^5}$ by means of power series expansions and by means of l'Hôpital and compare the amount of work for both. (You may choose to move $\cos x$ into the denominator for a possibly simpler calculation.)

Hwk 20: Calculate $\lim_{x\to 0} \frac{\sinh(\sin x) - \sin(\sinh x)}{x^7}$. Don't even think of using l'Hôpital, unless you have lots of time to kill.

Hwk 21: Find a power series $g(z) = a_1 z + a_2 z^2 + a_3 z^3 + \ldots$ such that $e^{g(z)} - 1 = z$. Calculate the coefficients practically up to a_4. You may see a pattern arising that you may expect from real-variable calculus already, but you do not need to prove a general formula for a_n; the main purpose is to do practical calculations with power series.

Hwk 22: "You can count on power series".
George Pólya has shown that there exists a holomorphic function $f(z)$ expressed in terms of a power series $f(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3 + \ldots$ that satisfies the equation

$$f(z) = 1 + \frac{z}{6}\left(f(z)^3 + 3f(z)f(z^2) + 2f(z^3)\right)$$

Calculate $a_0, \ldots a_6$ from this. To help you guard against miscalculations, I reveal to you that all the a_n are positive integers. If by a_5 you believe to see a pattern, you may be generalizing too fast. a_6 doesn't confirm your conjecture.

Comment, just for fun: I know that all a_n are integers, because George Pólya showed in the same context that the a_n actually describe the number of mathematically conceivable isomeric structures corresponding to the chemical formula $C_nH_{2n+1}OH$. Not all of them are expected to be chemically stable. — While the case $n = 2$ is banned from dry campuses, the sober-minded mathematical accounting thereof is not;-) The case $n = 1$ can be blinding, but the mathematical accounting thereof still keeps a clear view.

The connection between the power series and its subtle counting property is covered in Math 521-522.

Hwk 23: If an analytic function satisfies $f'(z) = 1 + f(z)^2$ and $f(z) = 0$, then find the terms of its power series up to order z^7. (This is another way of calculating Taylor coefficients for $\tan z$, since $f(z) = \tan z$ is actually the analytic function satisfying the given hypotheses.)

From book: pg 152 #16