Sample problems Solutions sections 2.3 & 2.4.

1) Your company estimates it will have to replace a piece of equipment at a cost of $800,000 in 5 years. To do this a sinking fund is established by making equal monthly payments into an account paying 6.6% compounded monthly. How much should each payment be? ($11,290.42)

$$A=\frac{P\left(\left(1+\frac{r}{m}\right)^{mt}-1\right)}{r/m} \quad \Rightarrow \quad 100000=\frac{P\left(\left(1+\frac{0.066}{12}\right)^{60}-1\right)}{0.066/12}$$ solving for P gives

P = $11,290.42

2) Betty deposits $2000 annually into a Roth IRA that earns 6.85% compounded annually. Due to a change in employment, these deposits stop after 10 years, but the account continues to earn interest until Betty retires 25 years after the last deposit is made. How much is in the account when Betty retires? ($143,785.10)

First determine the accumulation from the periodic deposits:

$$S=\frac{P\left(\left(1+\frac{r}{m}\right)^{mt}-1\right)}{r/m} = \frac{2000\left(\left(1+0.0685\right)^{10}-1\right)}{0.0685}=27,437.89.$$ *Now this amount earns interest for 25 years compounded annually:*

$$A=P\left(1+\frac{r}{m}\right)^{mt}=27437.89\left(1+0.0685\right)^{25}=143,785.10.$$

3) You make monthly deposits of \$100 into an annuity and after 30 years wish to accumulate \$160,000. What annual rate compounded monthly will be required to do this? (0.083480405763)

Here in $S=\dfrac{P\left(\left(1+\frac{r}{m}\right)^{mt}-1\right)}{r/m}$ we sill solve for r:

$$160000=\frac{100\left(\left(1+\frac{r}{12}\right)^{360}-1\right)}{r/12}\quad\Rightarrow\quad r=0.083480405763$$

4) You desire to save \$200,000 for retirement. You can afford to save \$125 a month into a mutual fund that averages7.75% compounded monthly. How many years will be needed to do this? (31.426831333098)

We need to solve $S=\dfrac{P\left(\left(1+\frac{r}{m}\right)^{mt}-1\right)}{r/m}$ for t.

$$200000=\frac{125\left(\left(1+\frac{0.0775}{12}\right)^{12t}-1\right)}{0.0775/12}\qquad\Rightarrow\quad t=31.426831333098\,.$$

5) You decide to buy a TV set for \$800 and agree to pay for it with 18 equal monthly payments at 1.5% interest per month on the unpaid balance. How much are your payments? (\$51.05) What is the total interest paid? (\$118.90)

Here we use the present value of an annuity formula: $V=\dfrac{P\left(1-\left(1+\frac{r}{m}\right)^{-mt}\right)}{r/m}$

$$800=\frac{P\left(1-\left(1+\frac{0.18}{12}\right)^{-18}\right)}{0.18/12}\quad\Rightarrow\quad P=51.05$$

The total interest is the total paid – initial cost: 18(51.05)-800 = 118.90.

6) American Capital offers a 7-year ordinary annuity with a guaranteed rate of 6.35% compounded annually. How much should you pay for one of these annuities if you want to receive payments of $10,000 annually over the 7-year period? ($55,135.98)

We need the value of the annuity, V. Use $V = \dfrac{P\left(1-\left(1+\dfrac{r}{m}\right)^{-mt}\right)}{r/m}$ =

$$V = \frac{10000\left(1-\left(1+\dfrac{0.0635}{1}\right)^{-7}\right)}{0.0635/1} = 55135.98.$$

7) Al Bundy says he paid $25,000 down on a new house and will pay $525 per month for 30 years. If interest is 7.8% compounded monthly, what was the selling price of the house? ($97929.78)

First calculate the present value of the loan ($ borrowed)

$$V = \frac{P\left(1-\left(1+\dfrac{r}{m}\right)^{-mt}\right)}{r/m} = \frac{525\left(1-\left(1+\dfrac{0.078}{12}\right)^{-360}\right)}{0.078/12} = 72{,}929.78$$

Then add the down payment: 72929.78 + 25000 = 97929.78.

8) You have found the house of your dreams. The selling price is $175,000 with an interest rate of 5.5% compounded monthly. Determine the monthly house payment if the loan is for:

a) 30 years ($993.64)   b) 15 years ($1429.90)

a) $V = \dfrac{P\left(1-\left(1+\dfrac{r}{m}\right)^{-mt}\right)}{r/m} \quad \Rightarrow \quad 175000 = \dfrac{P\left(1-\left(1+\dfrac{0.055}{12}\right)^{-360}\right)}{0.055/12} \quad \Rightarrow \quad P = 993.64$

b) Changing the 360 to 180 gives P = 1429.90.

Determine the total interest paid for the loan in part (a) ($182,710.40) and (b) ($82,382).

Interest (a): 993.64(360) – 175000 = 182710.40.

Interest for (b): 1429.90(180)-175000 = 82382.

Suppose you have financed your home for 30 years. How much is the unpaid balance after making payments for 20 years? ($91,557.55)

This unpaid balance forms another annuity. The present value of this annuity will be the amount owed after making the 240th payment.

$$V=\frac{P\left(1-\left(1+\frac{r}{m}\right)^{-mt}\right)}{r/m}\quad\Rightarrow\quad V=\frac{993.64\left(1-\left(1+\frac{0.055}{12}\right)^{-120}\right)}{0.055/12}=91557.55.$$

Suppose before making the first payment you receive a raise and can pay an extra $150 each month (30 year loan). How long will it take to pay off the mortgage? (22.022274711642 years)

Instead of the regular payment of 993.64 we can pay 1143.64. Solving for t gives t = 22.022274711642 years.

$$V=\frac{P\left(1-\left(1+\frac{r}{m}\right)^{-mt}\right)}{r/m}\quad\Rightarrow\quad 175000=\frac{1143.64\left(1-\left(1+\frac{0.055}{12}\right)^{-12t}\right)}{0.055/12}.$$

9) At the time of retirement, a couple has $200,000 in an account that pays 8.4% compounded monthly. If they decide to withdraw equal monthly payments for 10 years, at the end of which time the account will have zero balance, how much should they withdraw each month? ($2469.04)

$$V=\frac{P\left(1-\left(1+\frac{r}{m}\right)^{-mt}\right)}{r/m}\quad\Rightarrow\quad 200000=\frac{P\left(1-\left(1+\frac{0.084}{12}\right)^{-120}\right)}{0.084/12}\quad\Rightarrow\quad P=2469.04.$$

10) Two twins Lauren & Mallory both will save $2000 at 12% compounded annually. Mallory begins at age 20 and deposits $2000 a year till age 29, for a total of 10 deposits, then does nothing till retirement at age 65 (36 years). How much will Mallory have at age 65? Lauren begins at age 29 depositing $2000 a year until retirement at age 65 (37 deposits). How much will Lauren have at retirement? (Mallory: $2,075,509.03) (Lauren: $1,087,197.38).

Mallory: *First determine the accumulation of the 10 deposits.*

$$A = \frac{P\left(\left(1+\frac{r}{m}\right)^{mt} - 1\right)}{r/m} = \frac{2000\left((1+0.12)^{10} - 1\right)}{0.12} = 35097.47$$ *then this is compounded annually for 36 years* $\Rightarrow$ $A = 35097.47(1 + 0.12)^{36} = 2{,}075{,}509.03$.

Lauren: $$A = \frac{P\left(\left(1+\frac{r}{m}\right)^{mt} - 1\right)}{r/m} = \frac{2000\left((1+0.12)^{37} - 1\right)}{0.12} = 1{,}087{,}197.38.$$

AND LAUREN WILL ***<u>NEVER</u>*** CATCH MALLORY.

START SAVING **<u>EARLY</u>**!!!!!!!!!!!!!!!!