Triangle Angle: In a triangle T formed by circles of radii x, y, z, the angle at the circle of radius x is given by the following, depending on geometry:

Euclidean:

$$\alpha(x; y, z) = \arccos\Big(\frac{(x+y)^2 + (x+z)^2 - (y+z)^2}{2(x+y)(x+z)}\Big)$$

Spherical: (side condition: $x + y + z \leq \pi$)

$$\alpha(x; y, z) = \arccos\Big(\frac{\cos(y+z) - \cos(x+y)\cos(x+z)}{\sin(x+y)\sin(x+z)}\Big)$$

Hyperbolic:

$$\alpha(x; y, z) = \begin{cases} \arccos\Big(\dfrac{\cosh(x+y)\cosh(x+z) - \cosh(y+z)}{\sinh(x+y)\sinh(x+z)}\Big), & x, y, z \in (0, \infty) \\ \arccos\Big(\dfrac{\cosh(x+y) - e^{(y-x)}}{\sinh(x+y)}\Big), & x, y \in (0, \infty), z = \infty \\ \arccos\big(1 - 2e^{2x}\big), & x \in (0, \infty), y = z = \infty \\ \\ 0 & x = \infty. \end{cases}$$

Angle Sums: In a flower having central label r and petal labels $\{r_1, \cdots, r_k\}$, the angle sum is given by the following summation formula, where $m = k$ and $r_{k+1} = r_1$ if the flower is closed, and $m = k - 1$ otherwise:

$$\theta(r; r_1, \cdots, r_k) = \sum_{j=1}^{m} \alpha(r; r_j, r_{j+1}).$$