December 4th, 2008

# Math 300

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Fall 2007

Name:..........................................

Student ID (last 6 digits): XXX- ....................

## Final

You have two hours to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 10 questions and 12 printed pages (including this one and two pages for scratch work in the end).

No books, notes or calculators are allowed on this exam!

**Show all work!** Even correct answers without work may result in point deductions. Also, **points will be taken from messy solutions**.

**Good luck!**

| Question | Max. Points | Score |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 10 | |
| 8 | 10 | |
| 9 | 10 | |
| 10 | 10 | |
| Total | 100 | |

**1)** Negate the following statement: *"given* $x \in (0, \infty)$*, we have that* $x > y > 0$ *for some* $y \in (0, \infty)$*"*. [As usual, your answer should contain no "nots".]

**2)** Let $A, B, C$ be subsets of $U$. [You can use $U = \mathbb{R}$ if you want, but it is not necessary.] Prove of disprove: $(A \setminus C) \cup (A \cap B) = A \cap ((B \cap C)^{\mathrm{c}})$.

**3)** Let $\sim$ be the relation on $\mathbb{R}^2 \setminus \{(0,0)\}$ defined by $(a,b) \sim (c,d)$ iff there exists $r \in \mathbb{R}$ such that $(ra, rb) = (c,d)$. Prove that $\sim$ is an equivalence relation.

**4)** Let $f : X \to Y$ be a one-to-one function and $A, B \subseteq X$. Prove that $f(A \cap B) = f(A) \cap f(B)$.

**5)** Show that for all positive integers $n$, we have that $n^7 - n$ is divisible by 7.

**6)** Prove that $n! \leq n^n$ for all positive integers $n$.

**7)** Find a closed formula for the recursion $a_0 = 0$, $a_n = 3 \cdot a_{n-1} + 2$ for $n \geq 1$. [You don't have to show me how you came up with the formula, but you have to prove that it is correct.]

**8)** Let $F$ be a field and $a, b \in F$. Also, let $n(x)$ denote the additive inverse of $x$ [which I denoted by $-x$] and $q(x)$ denote the multiplicative inverse of $x$ [which I denoted by $x^{-1}$]. *Using only the field axioms* show that:

(a) $n(a + b) = n(a) + n(b)$

(b) $q(q(a)) = a$

**9)** Let $F$ be an ordered field. Using only the order axioms show that if $0 < a < b$ and $0 < c < d$, then $ac < bd$.

**10)** Let $x \in \mathbb{R}$ such that $|x-1| < 2$. Show that $\left|x^2 - 2x + 2\right| < 5$.

**Scratch:**

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