September 30th, 2009

Math 300

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Fall 2009

Name:...

Student ID (last 6 digits): XXX- ....

Midterm 1

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 5 questions and 7 printed pages (including this one and a page for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work! Even correct answers without work may result in point deductions. Also, **points will be taken from messy solutions**. Remember you also be graded on how well written your proofs are.

Good luck!

Question	Max. Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1) [20 points] Mark true or false. Justify your answers only for the ones which are false. [No need to justify if true.]

(a) $\mathcal{P} = \{\{1, 2, 3, 4, 5\}\}$ is a partition of $\{1, 2, 3, 4, 5\}$.

(b) $\mathcal{P} = \{\{1\}, \{2, 5\}, \{3, 4\}\}$ is a partition of $\{1, 2, 3, 4, 5, 6\}$.

(c) Let X be the set of all people in the world, and define the relation $\mathcal{R}$ by $x\mathcal{R}y$ if x is married to y. Then, $\mathcal{R}$ is an equivalence relation.

(d) Let X be the set of all people in the world, and define the relation $\mathcal{R}$ by $x\mathcal{R}y$ if x and y have the same mother. Then, $\mathcal{R}$ is an equivalence relation.

(e) The converse of an "if-then" statement is logically equivalent to the original statement [i.e., either both are false or both are true].

2) [20 points] Give the completely simplified negation of the following statement. [Your answers should have no "nots" in them.]

For all $\epsilon > 0$, *there is* $\delta > 0$ *such that either* $|x| \geq \delta$ *or* $|x^2| < \epsilon$.

[**Hint:** Negate one part at a time, as it makes it easier to get partial credit.]

3) [20 points] Prove that if A and B are symmetric subsets of $\mathbb{R}$, then so is $A \cap B$.

4) [20 points] Let $\mathcal{R}$ be the relation on $\mathbb{R}^2$ given by $(x_1, y_1)\mathcal{R}(x_2, y_2)$ if there exists $r \in \mathbb{R}$ such that $(x_2, y_2) = (x_1 + r, y_1 + r)$.

[So, be careful here! You will use ordered pairs for elements! I.e., use "assume $(x_1, y_1)\mathcal{R}(x_2, y_2)$" instead of "assume $x\mathcal{R}y$" in your proofs.]

(a) Prove that $\mathcal{R}$ is an equivalence relation.

(b) Give the equivalence class of $(0, 0)$.

5) [20 points]

(a) Prove that $x \notin A \setminus C$ can be interpreted as saying that $x \notin A$ or $x \in C$. [You need to give a *formal* proof! Wordy arguments will receive partial credit at best. **Hint:** Remember how to negate "and/or" statements.]

(b) Let A, B and C be sets. Prove that $(B \cap C) \cup (B \setminus A) = B \setminus (A \setminus C)$. [**Hint:** Use item (a). Note that you can use it even if you did not do that part!]

Scratch: