November 4th, 2009

Math 300

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Fall 2009

Name: .....

Student ID (last 6 digits): XXX- .....

Midterm 2

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 5 questions and 7 printed pages (including this one and a page for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work! Even correct answers without work may result in point deductions. Also, **points will be taken from messy solutions**. Remember you also be graded on how well written your proofs are.

Good luck!

Question	Max. Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1) [20 points] Mark true or false. Justify your answers only for the ones which are false. [No need to justify if true.]

(a) For every function $f : X \to Y$ and $A, B \subseteq X$, we have that $f(A \cap B) = f(A) \cap f(B)$.

(b) For every function $f : X \to Y$ and $A, B \subseteq Y$, we have that $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$.

(c) The function $f : \mathbb{R} \setminus \{0\} \to \mathbb{R} \setminus \{0\}$ defined by $f(x) = 1/x^2$ is one-to-one.

(d) The function $f : \mathbb{R} \setminus \{0\} \to \mathbb{R} \setminus \{0\}$ defined by $f(x) = 1/x^2$ is onto.

(e) If $f : X \to Y$ is invertible and $A, B \subseteq X$, then both $f^{-1}(f(A)) = A$ and $f(A \setminus B) = f(A) \setminus f(B)$ are true.

2) [20 points] Functions:

(a) Let $f : \mathbb{R} \to R$ be $f(x) = x^2$. Give $f([-2, 3])$ and $f^{-1}((-1, 3))$.

(b) Is $g : \mathbb{R} \setminus \{0\} \to \mathbb{R} \setminus \{0\}$ given by $g(x) = 1/x$ invertible? [Don't forget to justify!]

3) [20 points] Prove *by induction* that for $n \in \mathbb{N}$:

$$\sum_{k=1}^{n}(2k+1) = n^2 + 2n.$$

4) [20 points] Prove that $3^{2n} - 1$ is divisible by 8 for all $n \in \mathbb{N}$.

5) [20 points] Prove that $n + 2 \leq 3^n$ for all integers $n \in \mathbb{N}$.

Scratch: