Dec 8, 2005

Math 151

Autumn 2005

Luis Finotti

Name: ....

SSN (last 4 digit): XXX-XX-................

Check Your Recitation Class:

Rich Hambrock	Craig Lennon
☐ 10:30	☐ 10:30
☐ 11:30	☐ 11:30

Final Exam *Form A*

• You have 108 minutes to answer the following 7 questions. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

• Write your name and the last four digits of your social security number on the top of this page. and check off your recitation section by time and TA.

• Check that no pages of your exam are missing. This exam has 12 printed and numbered pages.

• No books or notes are allowed on this exam! Calculators are permitted only as stated in the "Calculator Policy" of the course. Electronic communication devices of any kind are prohibited.

• Show all work! Even correct answers without work may result in point deductions. Do not simplify arithmetical expression such as $\frac{\sqrt{7}}{3} - \frac{5}{\sqrt{11}}$.

• Plan your time wisely! It is a good idea to skim over all of the questions before you start. Do not spend too much time on one particular problem.

Good Luck !

Question	Max. Points	Score
1	24	
2	36	
3	26	
4	26	
5	28	
6	32	
7	28	
Total	200	

1) [24 Points] **You must show all work!**

Find the following limits. Your answer should be either a number, "$+\infty$", "$-\infty$", or *"DNE"*

(a) $\displaystyle\lim_{t\to 3^-} \frac{t^3 - 3t^2}{t^2 + t - 12} =$

(b) $\displaystyle\lim_{\nu\to 0} \frac{\sin(7\nu)}{\tan(5\nu)} =$

(c) $\displaystyle\lim_{u\to -\infty} \frac{9 + 2u^5}{\sqrt{7u^{10} + 8}} =$

2) [36 Points]

Show all work! Do not simplify expressions!

Find the following:

(a) Compute $\dfrac{d}{dt}\left(\dfrac{\sin(t) + 4t^9}{\sqrt[8]{t} + \sqrt{9}}\right) =$

(b) For $f(x) = \cos(\sqrt{\tan(x)})$ compute $f'(x) =$

Show all work! Do not simplify expressions!

(c) Suppose $h(z) = z^{-\frac{3}{2}} \cdot g(z) + g(\sqrt{z})$, where the function $g(z)$ has values:

$g(2) = \sqrt{11}$, $g'(2) = -13$, $g(4) = \frac{1}{7}$, and $g'(4) = 3$,

Find the value of $h'(4) =$

You need not simplify numerical expressions.

(d) Find the following antiderivative: $\displaystyle\int \frac{2}{t^6} + \sin(t) + 15t^2 \sqrt[6]{(5t^3+8)}\ dt$.

3)[26 Points] **Show all Work!**

The curve described by the following equation between x and y is a tilted ellipse in the xy-plane centered at the origin:

$$5x^2 + 6xy + 3y^2 = 6$$

(a) Find an expression in x and y for $\frac{dy}{dx}$ using implict differentiation.

(b) Find all points on the ellipse at which the slope of the tangent is $m = -1$.

4) [26 Points] Consider the following function defined on $[0, \frac{\pi}{2}]$.

$$f(x) = 48 \cdot \cos(x) + 12 \cdot x^2 + (24\sqrt{3} - 8 \cdot \pi) \cdot x \ .$$

(a) Find the open intervals in $[0, \frac{\pi}{2}]$ on which $f(x)$ is concave up.

(b) Find the open intervals in $[0, \frac{\pi}{2}]$ on which $f(x)$ is concave down.

(c) Find the slope of the tangent line at one inflection point.

(d) Show explictly that $x = \frac{\pi}{3}$ is critical point of $f(x)$.

(e) Use previous answers to decide whether $x = \frac{\pi}{3}$ is a local maximum, local minumum, or neither. **Justify you answer!**

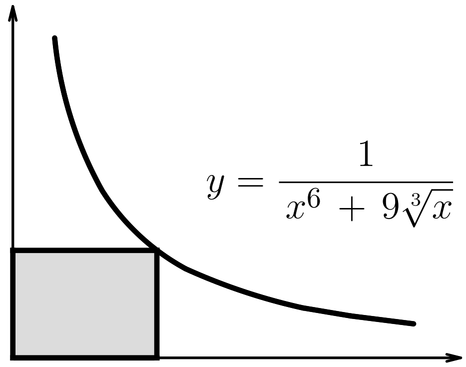
Do not simplify numerical expressions!

5)[28 Points] **Show all work!**

A rectangle is to placed above the x-axis, to the right of the y-axis, and below the graph of the function

$$f(x) = \frac{1}{x^6 + 9\sqrt[3]{x}}$$

See the diagram on the right.



What is the width of such a rectangle for which the area is maximal?

Your solution has to include the following:

(a) The area A as an expression in the coordinates x and y of the corner point on the curve.
(b) The area as a function $A = A(x)$ of only x.
(c) The domain of the function $A(x)$.
(d) The critical points of $A(x)$. **Give exact numerical expression, do not simplify!**
(e) The point where the area attains its global maximum.
(f) A precise argument why this is a global maximum.

6)[32 Points] Draw in the grid on the following page the graph of one function $f(x)$, which is *consistent* with all of the following properties:

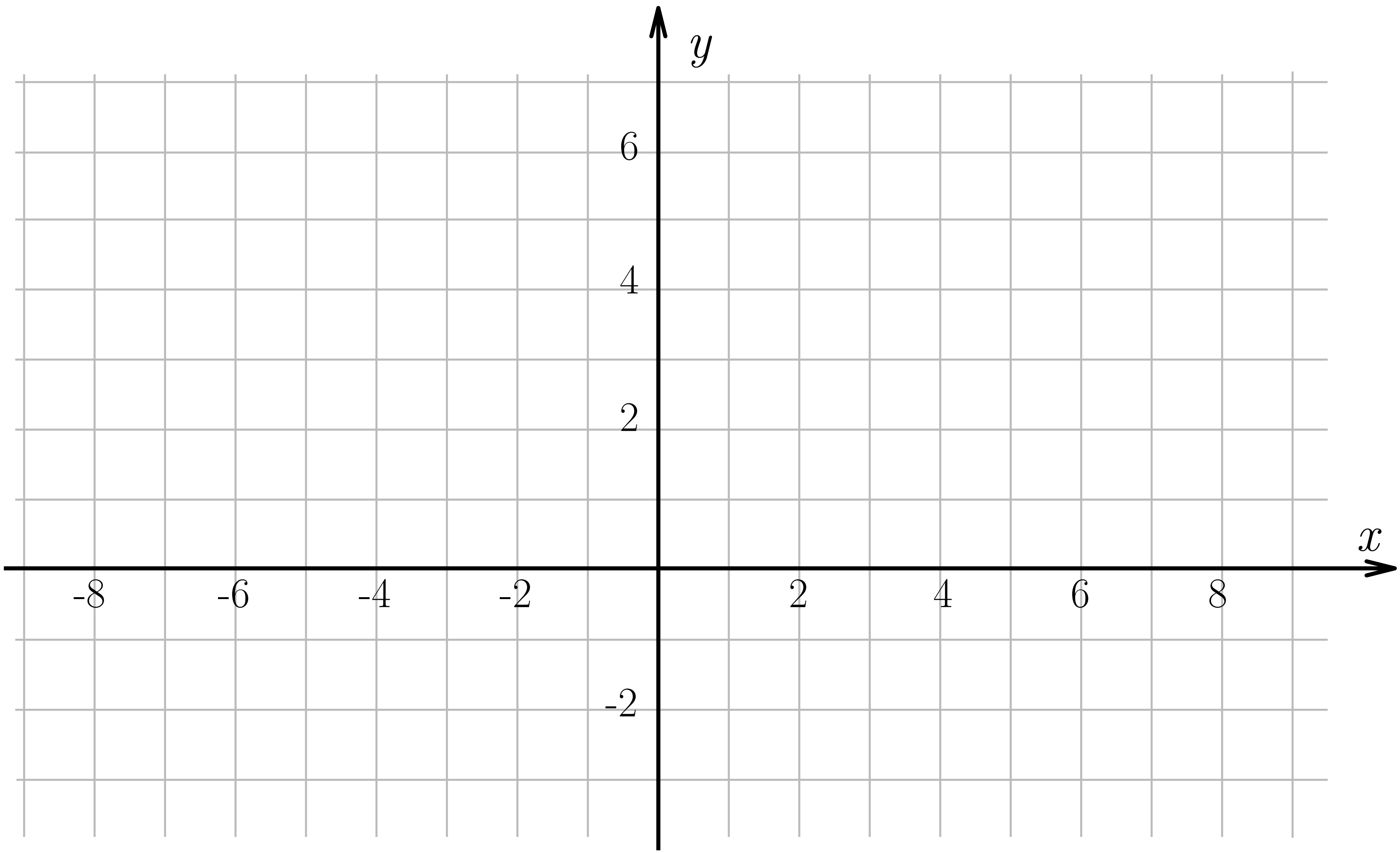
- $f(x)$ has domain $(-\infty, -2) \cup (-2, \infty)$ on which it is (twice) differentiable.
- $f(0) = -2$ and $f(-6) = 3$.
- $f'(4) = 2$, $f'(0) = 0$, and $f'(-6) = 0$.
- $f''(4) = 0$ and $f''(-6) = 0$.
- $f'(x) > 0$ on $(0, \infty)$
- $f'(x) < 0$ on $(\infty, -6) \cup (-6, -2) \cup (-2, 0)$
- $f''(x) < 0$ on $(-6, -2) \cup (4, \infty)$
- $f''(x) > 0$ on $(-\infty, -6) \cup (-2, 4)$
- $\lim_{x \to -2^-} f(x) = -\infty$ and $\lim_{x \to -2^+} f(x) = +\infty$
- $\lim_{x \to +\infty} f(x) = 3$

You must show how the graph looks for all x in $(-9, -2) \cup (-2, 9)$.

The graph has to show clearly all monotonicity and concavity properties of $f(x)$.

Besides the graph itself your drawing must contain the following clearly drawn and indicated lines:

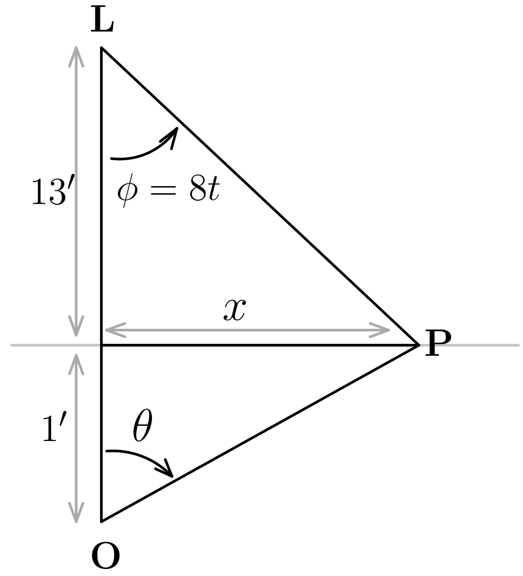
- The tangent lines at all critical points.
- The tangent lines at all inflection points.
- All vertical asymptotes.
- All horizontal asymptotes.


y
6
4
2
x
-8
-6
-4
-2
2
4
6
8
-2

7) [28 Points]

A rotating light source at a point **L** is positioned 13 feet from a screen. The light beam hits the screen at a spot **P** and has an angle ϕ with the shortest line from **L** to the screen. The angle changes uniformly with time as (in units 'radians' and 'seconds')

$$\phi = 8t \qquad \text{for } t \text{ in } [0, \tfrac{\pi}{16}].$$



On the other side of the screen directly across from the light and one foot away from the screen is an observer at point **O** looking at the light spot **P**. The line of sight has an angle θ with the shortest line between **O** and the screen. See the picture on the right.

Find an explicit expression in t for the rate $\dfrac{d\theta}{dt}$ by which the observer angle changes. Particularly, answer the following questions:

(a) Find a relation between the indicated distance x and ϕ as well as a relation between x and θ.

(b) Use (a) to find a relation between θ and ϕ only. [*Hint:* Substitute for x]

(c) Find a relation involving θ, ϕ, $\frac{d\theta}{dt}$ and $\frac{d\phi}{dt}$. [*Hint:* Differentiate the equation you found in (b)]

(d) Find a relation between ϕ, $\frac{d\theta}{dt}$ and $\frac{d\phi}{dt}$ only. To this end use (b) to substitute for any expressions in θ occurring in the equation you found in (c). [*Hint:* $\sec^2(\theta) = 1 + \tan^2(\theta)$]

(e) Solve for $\frac{d\theta}{dt}$ to find an expression for $\frac{d\theta}{dt}$ in ϕ and $\frac{d\phi}{dt}$ only.

(f) Use the explicit expression for ϕ in t in order to write $\frac{d\theta}{dt}$ as a function in t only.

(g) Is there a time $t = c$ in $(0, \frac{\pi}{16})$ for which $\dfrac{d\theta}{dt} = 8$? **Justify your answer using theorems from lecture!**

Additional Workspace:

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