May 7th, 2012

Math 141

Name: .....

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Spring 2012

Student ID (last 6 digits): XXX- .....

TA recitation (check one):

Chelsea McAmis:	☐	Joshua Mike:	☐
Tim Krumwiede (11:15):	☐	Tim Krumwiede (12:40):	☐

Final

You have two hours to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 9 questions and 14 printed pages (including this one and two pages for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	10	
2	15	
3	15	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
Total	100	

1) Let $f(x) = \dfrac{(x-1)^2(x+2)^2}{x^2-3x+2}$.

(a) [2 points] Give the domain of $f(x)$.

(b) [3 points] Give the values of x for which we have $f(x) = 0$.

(c) [5 points] Give the *intervals* where $f(x) < 0$.

2) Compute the following limits.

(a) [5 points] $\lim_{x \to 1} \dfrac{x^3 + \mathrm{e}^x}{x^2 - 2x + 1}$

(b) [5 points] $\lim_{x \to -\infty} \dfrac{2x^3 - 3x + 1}{x^3 + \sqrt[3]{x}}$

(c) [5 points] $\lim_{x \to 0} \dfrac{\cos(2x^2) - \mathrm{e}^x}{3\tan(3x)}$

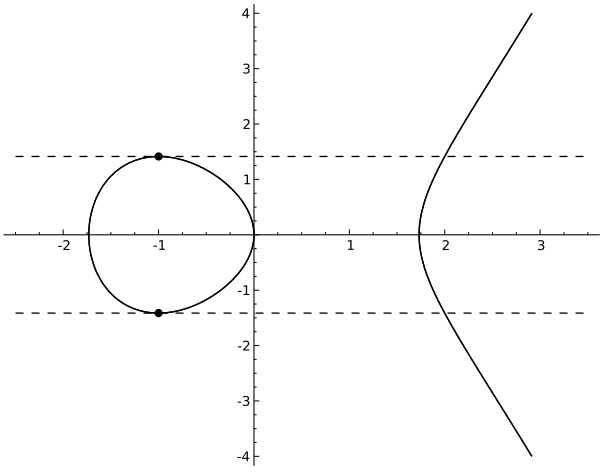
3) Compute the following derivatives:

(a) [5 points] $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\arctan(2^x)\right)$

(b) [5 points] $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\ln(x)\sqrt{x}\right)$

(c) [5 points] $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{f(g(x))}{f(x)g(x)}\right)$ [your answer should be a formula in terms of $f(x)$, $g(x)$, $f'(x)$ and $g'(x)$]

4) [10 points] Find the coordinates [x and y-coordinates] of the points with horizontal tangent line on the curve given by $y^2 = x^3 - 3x$.
[**Note:** The graph is given below, but you *cannot* use it in the solution. It might be useful to verify if your answer seems correct, though.]

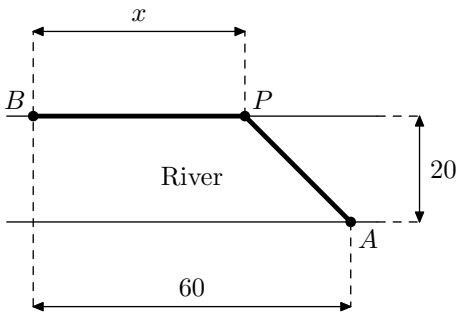


5) [10 points] Find the maximum and minimum of $f(x) = 2x^3 - 15x^2 + 24x + 7$ on $[0, 6]$ as well as the values of x in which they occur.

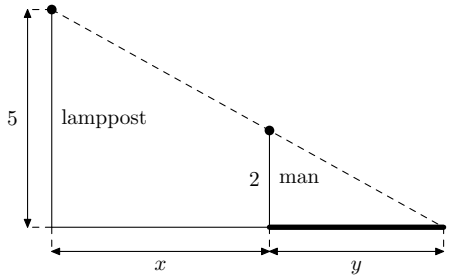
6) [10 points] [In this question you will set up, *but not solve* an optimization problem.]

A person can swim at a speed of 3 feet per second and run at a speed of 5 feet per second. He has to cross a 20 feet long river from point A [see picture below] to get to point B which is 60 feet to the left of the opposite margin of point A.

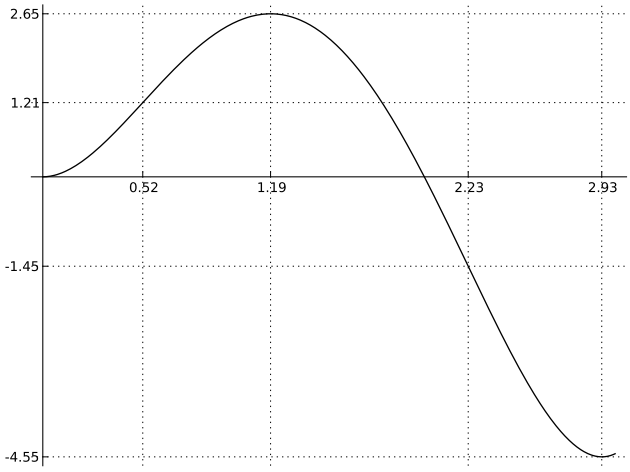
Find a function that gives the *time* it will take the person to go from A to B if he swims first to a point x feet to the right of B [labeled P in the picture] and then runs the rest of the way. [This function should involve x only!] Also, give a closed interval for the values of x in which the minimum of this function would give us the minimum time to go from A to B.



7) [10 points] A two meter tall man walks away from a five meter high lamppost at a speed of 3 meters per second. [See picture below.] How fast is the length of the shadow [denote by y in the picture] increasing?



8) [10 points] The *position* of a particle moving along a straight line at time t is given by $s(t)$. The graph of $s(t)$ for t in $[0, 3]$ is given below. [Coordinates of all local maxima/minima and inflexion points are given in the graph. Note that the tangent line at $t = 0$ is *horizontal*!]



Answer the following based on the graph. [No need to justify these.]

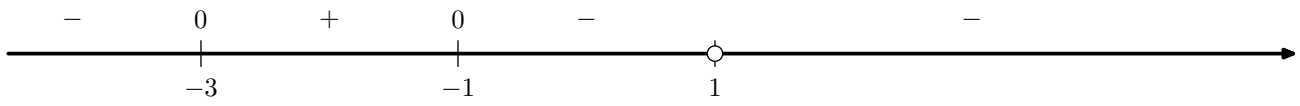
(a) At what time(s) is the *velocity* [not position!] maximal and at what time(s) is it minimal?

(b) At what time(s) was the velocity equal to zero?

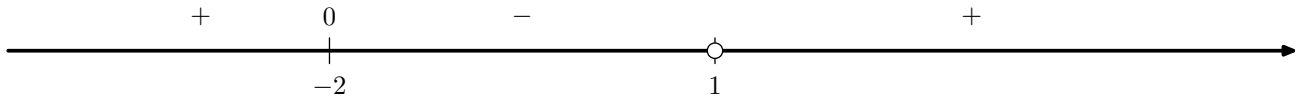
(c) When was the *acceleration* [not position, nor velocity!] negative? Give your answer as an interval.

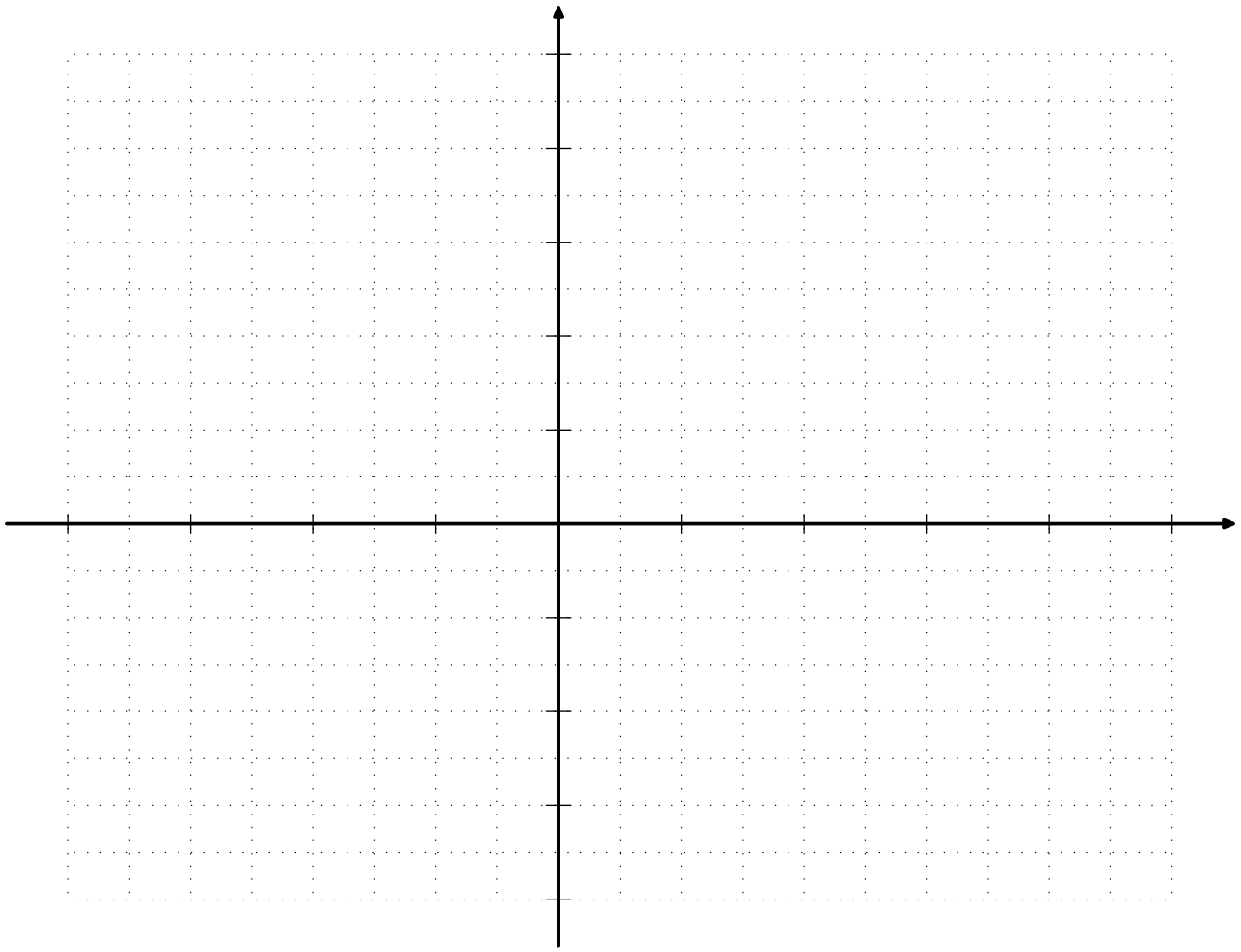
9) [10 points] Sketch the graph of a function $f(x)$ which satisfies all of the following conditions [draw concavities carefully!]:

- domain is all real numbers except 1;
- x-intercepts are -3.5, -2.5, 0, and y-intercept is 0;
- $f(-3) = -1$, $f(-1) = 2$;
- $\lim_{x\to-\infty} f(x) = \infty$, $\lim_{x\to\infty} f(x) = 0$, $\lim_{x\to1_-} f(x) = -\infty$, $\lim_{x\to1_+} f(x) = \infty$;
- the sign of the derivative is given by:



- the sign of the second derivative is given by:





Scratch: