Midterm

M552 – Modern Algebra II

March 12th, 2012

Solve as many as you can in class. [I would hope you could do all of this in class, as it is comparable to one part of the prelim.] Then, you may take this sheet home and solve all remaining problems, or problems that you think you've missed, and bring it to class on Wednesday. I will consider it for some partial credit. [At *most* half of the original number of points you've missed in the question.]

You should treat these problem as a *take-home* exam, not as a homework. So, you should not discuss *anything* about these problems with *anyone*. You can, however, use your book and notes. [No internet or other textbooks, though.]

We assume that R is a commutative ring with $1 \neq 0$.

1. [50 points] An R-module is called *artinian* if it satisfies the descending chain condition for submodules.

Suppose L, M and N are R-modules yielding the short exact sequence:

$$0 \longrightarrow L \xrightarrow{\psi} M \xrightarrow{\phi} N \longrightarrow 0$$

Show that if L and N are artinian, then so is M.

[**Note:** The converse is also true and easier to prove.]

2. [50 points] Let M and N be R-modules and M' and N' be submodules of M and N respectively. Define L as the sumbodule of $M \otimes_R N$ generated by the set $\{x \otimes y \in M \otimes_R N \ : \text{either } x \in M' \text{ or } y \in N'\}$. Show that $M/M' \otimes_R N/N' \cong (M \otimes_R N)/L$.

[**Note:** If the proof is straightforward, you can just say that a map is bilinear without proof.]