

# Math 351

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Spring 2015

Name: .....

Student ID (last 6 digits): XXX- .....

## MIDTERM 1 (IN CLASS)

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 6 questions and 8 printed pages (including this one and a page for scratch work in the end).

No books or notes are allowed on this exam!

**Show all work!** (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

**Good luck!**

Question	Max. Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
Total	72	

1) [12 points] Use the *Extended Euclidean Algorithm* to write the GCD of 87 and 51 as a linear combination of themselves. *Show the computations explicitly!* [**Hint:** You should get 3 for the GCD!]

**2)** [12 points] Find the remainder of the division of  $3^{222}$  when divided by 7 [i.e., what is  $3^{222}$  congruent to modulo 7]. *Show your computations explicitly!*

**3)** [12 points] Give the set of all solutions of the system

$$2x \equiv 4 \pmod{5}$$

$$x \equiv 3 \pmod{13}$$

4) [12 points] If we have that

$$7^{12} \equiv 1 \pmod{720}$$

then, what is the remainder of the division of  $7^{122}$  when divided by 720?

**5) LCM and GCD:**

(a) [6 points] Let  $a = 2^3 \cdot 5^4 \cdot 11$  and  $b = 3^2 \cdot 5^2 \cdot 7 \cdot 11$ . Find  $(a, b)$  [the GCD] and  $[a, b]$  [the LCM]. [You can leave powers and products indicated.]

(b) [6 points] If  $a = 14$ ,  $(a, b) = 7$  and  $[a, b] = 42$ , then what is  $b$ ? [Justify!]

**6)** [12 points] Prove that for all integers  $a$  and  $b$ , we have  $(a, b) = (a, a - b)$ .

**Scratch:**