June 11th, 2009

Math 251

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Summer 2009

Name: ....

Student ID (last 6 digits): XXX- ....

Midterm 1

You have 60 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 5 questions and 7 printed pages (including this one and a page for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	20	
2	15	
3	15	
4	25	
5	25	
Total	100	

1) [20 points] *Quickies!* [These should take you 10 seconds each if you've studied.] You don't need to justify your answers.

(a) If a square matrix is not invertible, what can we say about its reduced row echelon form? [Simple and short answer!]

(b) $$\begin{vmatrix} 2 & \sqrt{3} & -\pi & 12/131 \\ 0 & -1 & \mathrm{e}^2 & \ln(10) \\ 0 & 0 & 7 & \cos(3) \\ 0 & 0 & 0 & 1 \end{vmatrix} =$$

(c) $$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 5 \end{bmatrix} =$$

(d) If $E \cdot \begin{bmatrix} 2 & -1 & 3 \\ 0 & 2 & 0 \\ 5 & -4 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 3 \\ -5 & 6 & -1 \\ 5 & -4 & 1 \end{bmatrix}$, then $E =$

(e) If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 3$, then $\begin{vmatrix} g & h & i \\ d-g & e-h & f-i \\ 5a & 5b & 5c \end{vmatrix} =$

2) [15 points] Let

$$A = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 3 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 & -1 \\ 1 & 0 \\ 0 & 2 \end{bmatrix}.$$

Compute $\mathrm{tr}((A^{\mathrm{T}} \cdot B)^{-1})$.

3) [15 points] Let

$$A = \begin{bmatrix} 2 & 0 & 1 \\ -2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

Find $C_{2\,1}$, the cofactor of A at position $(2, 1)$ [or at $a_{2\,1}$, as the book writes] and $(\mathrm{adj}(A))_{1\,2}$, i.e., the entry at position $(1, 2)$ of the adjoint of A.

4) [25 points] Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}.$$

Find $\det(2 \cdot A^6)$ and A^{-1}. [Don't work harder than you have to!]

5) [25 points] Let

$$A = \begin{bmatrix} 2 & 4 & 1 & 0 \\ 1 & 2 & 0 & 0 \\ -1 & -2 & -1 & 1 \\ 4 & 8 & 2 & -1 \end{bmatrix}, \quad \mathbf{b}_1 = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 0 \\ -1 \\ -3 \\ -1 \end{bmatrix}$$

Solve the systems $A\mathbf{x} = \mathbf{b}_1$ and $A\mathbf{x} = \mathbf{b}_2$.

Scratch: